SENSITIVE ANALYSIS OF ROUGHNESS COEFFICIENT ESTIMATION USING VELOCITY DATA

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Abstract: *An accurate estimation of Manning's roughness coefficient n is of vital importance in any hydraulic study including open channel flows. In many rivers, the velocities at two-tenths and eight-tenths of the depth at stations across the stream are available to estimate Manning's roughness n based on a logarithmic velocity distribution. This paper re-investigates the method of the two-point velocity method and a sensitive analysis is theoretically carried out and verified with experiment data. The results show that velocity data can be used to estimate n for fully rough-turbulent wide channels. The results also indicate that the errors in the estimated n are very sensitive to the errors in x (the ratio of velocity at two-tenths the depth to that at eight-tenths the depth). The theoretical and experimental work shows that the smoother and deeper a stream, the more sensitive the relative error in estimated n is to the relative error in x.*

Keywords: open channels, roughness coefficient, two-point velocities, logarithm distribution.

1. INTRODUCTION

An accurate estimation of Manning's roughness coefficient n is of vital importance in any hydraulic study including open channel flows. This also has an economic significance. If estimated roughness coefficient are too low, this could result in over-estimated discharge, under-estimated flood levels and over-design and unnecessary expense of erosion control works and vice versa (Ladson et al., 2002).

The direct method to determine the value of roughness (Barnes, 1967, Hicks and Mason, 1991) is time consuming and expensive because friction slopes, discharges and some cross sections must be measured. Current practice many indirect or indirectly methods have been used to estimate roughness in streams from experience or some empirical relationship based on the particle size distribution curve of surface bed material (Chow, 1959, French 1985, Barnes, 1967, Hicks and Mason, 1991, Coon, 1998, Dingman and Sharma, 1997). However these methods are often applicable only to a narrow range of river conditions and the accuracy is still questionable.

In many rivers, a common method to measure stream flow is to measure velocity in several verticals at 0.2 and 0.8 times the depth with the velocity distribution depends on the roughness height. This may be related to Manning's n. For wide channels with reference to the logarithmic law of velocity distribution then the value of n can be determined based on this velocity data (Chow, 1959 and French, 1985). In practice, velocity measurement errors were unavoidable. In this paper, the two-point velocity method is re-investigate and a sensitive analysis is theoretically carried out and verified with experiment data.

2. THEORY

2.1 Relationship between velocity distrubution and roughness

The velocity distribution of uniform turbulent flow in streams can be derived by using Prandtl's mixing length theory (Schlichting, 1960). Based on this theory, the shear stress at any point in a turbulent flow moving over a solid surface can be expressed as:

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$$\tau = \rho l^2 \left(\frac{du}{dz} \right)^2 \quad (1)$$

where ρ is the mass density of the fluid, l is the characteristic length known as the mixing length ($l = \kappa z$,where κ is known as von Kármán's turbulent constant. The value of κ determined from many experiments is 0.4), u is velocity at a point, and z is the distance of a point from the solid surface.

The shear stress τ is equal to the shear stress on the bed τ_0 of the flow in the channel. From these two assumptions, Equation (1) can be written as

$$du = \frac{1}{\kappa} \sqrt{\frac{\tau_0}{\rho}} \frac{dz}{z} \quad (2)$$

Integrating Equation (2) gives

$$u = \frac{1}{\kappa} \sqrt{\frac{\tau_0}{\rho}} \ln \frac{z}{z_0} \quad (3)$$

where z_0 is the constant of integration.

It is also known that the bed shear stress τ_0 is represented as a bed shear velocity u_* defined by

$$u_* = \sqrt{\frac{\tau_0}{\rho}} \quad (4)$$

Thus Equation (3) can be written

$$u = \frac{u_*}{\kappa} \ln \frac{z}{z_0} \quad (5)$$

Equation (5) indicates that the velocity distribution in the turbulent region is a logarithmic function of the distance z. This is commonly known as the Prandtl-von Kármán universal velocity distribution law. The constant of integration, z_0, is of the same order of magnitude as the viscous sub-layer thickness. For natural channel, the flow is usually fully rough-turbulent, the viscous sub-layer is disrupted by roughness elements. The viscosity is no longer important, but the height of roughness elements becomes very influential in determining velocity profile. In this case z_0 depends only on the roughness height, usually expressed in terms of equivalent roughness k_s

$$z_0 = mk_s \quad (6)$$

where, in this case, m is a coefficient approximately equal to 1/30 for sand grain roughness (Keulegan 1938). Substituting Equation (6) for z_0 in Equation (5) yields

$$u = \frac{u_*}{\kappa} \ln \frac{30z}{k_s} \quad (7)$$

for mean velocity of turbulent flow for fully-rough flow in a wide channel (Keulegan,1938):

$$\frac{V}{U_*} = 6.25 + 2.5 \ln \frac{R}{k_s} \quad (8)$$

where V and U_* are cross-sectional mean velocity and shear velocity respectively and R is hydraulic radius.

In natural wide streams, the flow is usually fully rough-turbulent, and the logarithmic law of velocity distribution depending on the roughness height (Equations (7) and (8)) can be taken as the dominating factor that affects the velocity distribution. The roughness height and shear velocity are related to Manning's n. Hence, if this distribution is known, the value of Manning's n can be determined.

2.2 Two-point velocity method to estimate the value of Manning's n

Let $u_{0.2}$ be the velocity at two-tenths the depth, that is, at a distance 0.8D from the bottom of a channel, where D is the depth of the flow. Using Equation (7) the velocity may be expressed as

$$u_{0.2} = \frac{u_*}{\kappa} \ln \frac{24D}{k_s} \quad (9)$$

Similarly, let $u_{0.8}$ be the velocity at eight-tenths the depth, then

$$u_{0.8} = \frac{u_*}{\kappa} \ln \frac{6D}{k_s} \quad (10)$$

Eliminating u_* from the two equations above gives

$$\ln \frac{D}{k_s} = \frac{3.178 - 1.792x}{x - 1} \quad (11)$$

where $x = u_{0.2} / u_{0.8}$.

Substituting Equation (11) in Equation (8) for the rough channels with $R \approx D$ and simplifying yields

$$\frac{V}{U_*} = \frac{1.77(x+0.96)}{x-1} \quad (12)$$

Combining Manning's formula, $V = R^{2/3}\sqrt{S}/n$, and $U_* = \sqrt{gRS}$ (French, 1985) gives

$$\frac{V}{U_*} = \frac{R^{1/6}}{n\sqrt{g}} = \frac{D^{1/6}}{3.13n} \quad (13)$$

where D is in m, S is friction slope, and g is the gravitational acceleration ($g = 9.81 m/s^2$).

Equating the right-hand sides of Equations (12) and (13) and solving for n gives

$$n = \frac{(x-1)D^{1/6}}{5.54(x+0.96)} \quad (14)$$

This equation gives the value for Manning's n for fully-rough flow in a wide channel with a logarithmic vertical velocity distribution. It is suggested that when this equation is applied to actual streams, the value of D may be taken as the mean depth (Chow, 1959; French, 1985).

In practice, velocity measurement errors were unavoidable. The following section will investigate the affect of these errors on the estimated roughness using this method.

3. THEORETICAL SENSITIVITY ANALYSIS

Furthermore, considering errors of the roughness coefficient (Δn), depth (ΔD) and the ratio of two velocities (Δx) in the three quantities, to first order:

$$\Delta n = \frac{\partial n}{\partial D}\Delta D + \frac{\partial n}{\partial x}\Delta x \quad (15)$$

From Equations (14) and (15), the relationship between the relative error in n and the relative errors in D and x is obtained as

$$\frac{\Delta n}{n} = \frac{1}{6}\frac{\Delta D}{D} + \frac{1.96x}{(x+0.96)(x-1)}\frac{\Delta x}{x} \quad (16)$$

Equation (16) indicates that the relative error in n is always equal to 1/6 of the relative error in depth D, while it is expected to be more sensitive to the relative errors in x because of the term $(x-1)$ in the denominator.

In order to see the effect of errors in x on errors in the estimated n the relative errors in x are plotted against the relative errors in n with different values of depth and the roughness coefficient (see Figure 1). These relationships were calculated from the depth range of 0.5 m to 4 m and with a roughness coefficient range of 0.02 to 0.05. These are the common ranges of depth and the value of Manning's n in natural streams.

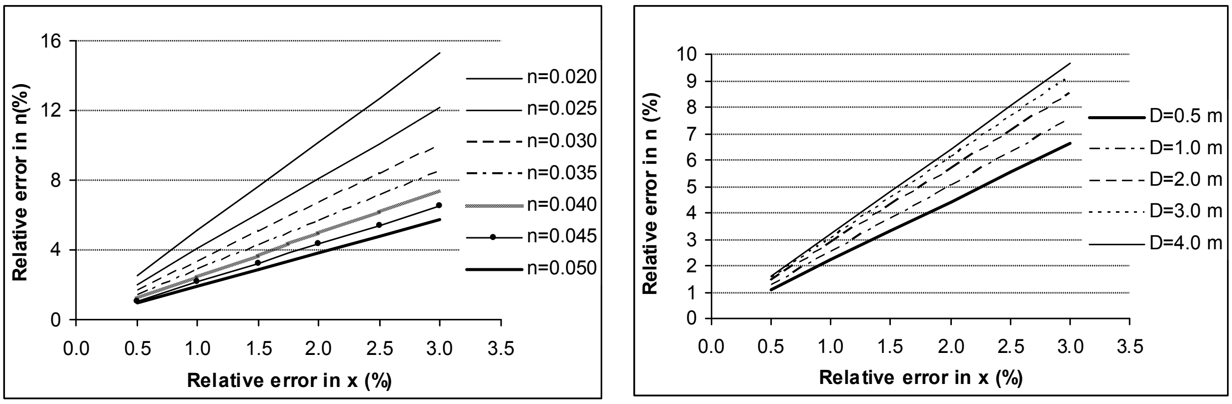


Figure 1. Relationship between relative errors in roughness n and relative errors in x (the ratio of velocity at 0.2 the depth to that at 0.8 the depth)

From these figures it can be seen that the relationship of the relative errors in n are very sensitive to the relative errors in x (the ratio of velocity at two-tenths the depth to that at eight-tenths the depth). The relative errors and relative errors in x depend on the depth and the roughness of streams. The smoother and deeper a stream is, the more sensitive the relative error in n is to the relative error in x. This indicates that the application of the two-point velocity method should be used with caution in relatively smooth deep rivers. However, this finding needs to be verified using the experiments that are discussed in the next section.

4. EXPERIMENTAL WORK AND ANALYSIS

4.1 Experimental equipment

The experimental runs were conducted in a laboratory flume in the Michell Hydraulic Laboratory, Department of Civil and Environmental Engineering at the University of Melbourne. The water was supplied to the flume from a constant head tank. Thus the supply always allowed steady conditions to be maintained. The inflow to the flume was controlled by a valve in the main supply line. Figure 2 shows the general arrangement of the experimental set-up.

The flume was 7100 mm long, 500 mm wide and 3800 mm deep. It was completely made of plexiglass and had an adjustable bed slope. Water entered to a turbulent suppression tank that was situated at the upstream end of the flume. A screen was provided inside the turbulent suppression tank near the entrance of this pipe to dampen the turbulence generated by the incoming flow into the tank.

The experiment was conducted using two different types of roughness. The first type of roughness is wire mesh with mesh size 6.5 mm square and the wire diameter of 0.76 mm. Such a method of roughening has been used in the past for simulating the bed roughness in free flow surface (e.g. Rajaratnam et al. 1976 and Zerihun 2004). A piece of mild steel wire screen The second type of roughness of the bed was gravel with the sieve analysis of $d_{50} = 16.5$ mm, $d_{84} = 19.5$ mm and $d_{90} = 20.0$ mm (see Figure 3).

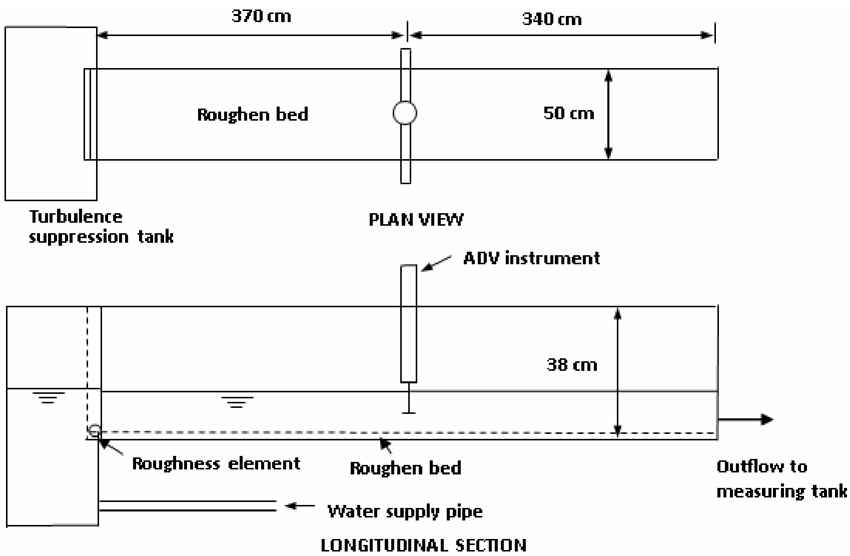


Figure 2. The experimental set-up diagram (not to scale)

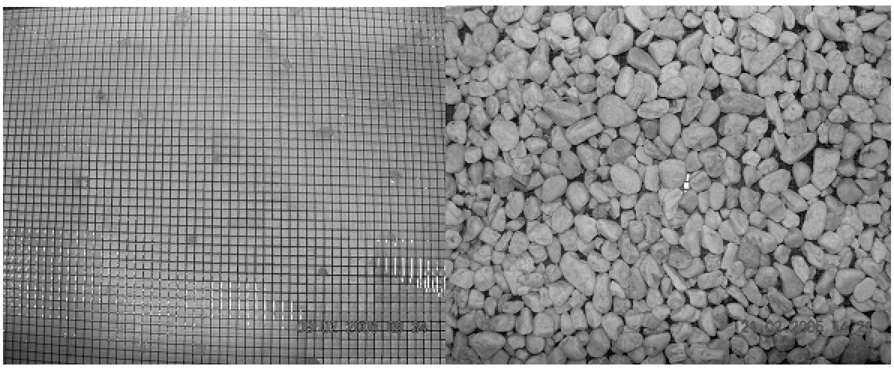


Figure 3. The two roughness types were carried out in the experiment

For all the tests the discharge was determined by a discharge measuring system. The vertical velocity profiles were measured by an Acoustic-Doppler Velocimeter (ADV) of a two-dimensional (2-D), side-looking probe manufactured by SonTek Inc.

The main objective of velocity profile measurement was to determine Manning's n by using the whole velocity profile and the two-point velocity method. For these purposes, velocity observations were done at closely spaced sections so that they could accurately describe the actual velocity profile. The duration of each velocity measurement was set between 60 and 65 s. Figure 3.6 show the velocity at z=1.7 cm of gravel bed with the water depth of 8.5 cm.

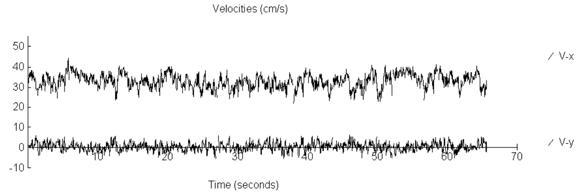


Figure 4. Velocity time series at $z = 1.7$ cm of the gravel bed flume of 8.5 cm water depth

4.2. Scope of the experiment

Eleven test runs were conducted for the ratio width/depth > 5 and fully rough turbulent flow with Reynolds number Re ranged from 15000 to 30000, the value of roughness Reynolds number Re_k ranged from 71 to 902 as shown in Table 1.

Table 1. Characteristic data of experimental runs

Surface type	Depth (cm)	Q (l/s)	V (cm/s)	Re	Re_k	Fr	n comp
Wire mesh $d_w = 0.76$ mm	6.4	13.70	42.81	19150	71.0	0.540	0.02186
	7.2	16.68	46.33	22472	73.4	0.551	0.02175
	7.5	17.85	47.59	23729	77.4	0.555	0.02168
	8.1	20.29	50.10	26279	79.0	0.562	0.02165
	8.5	21.93	51.60	27919	80.4	0.565	0.02160
	9.0	24.12	53.61	30072	82.5	0.571	0.02150
Gravel bed $d_{50} = 16.5$ mm	6.5	10.87	33.44	15124	772.2	0.419	0.02807
	7.0	12.34	35.76	16855	803.6	0.435	0.02794
	7.5	14.07	37.53	18714	838.2	0.438	0.02785
	8.0	15.90	39.27	20597	871.2	0.441	0.02773
	8.5	17.67	41.58	22500	902.6	0.455	0.02765

4.3. Results and discusion

For each test, firstly the whole velocity profiles were measured at every 2 or 3 mm intervals. Then the velocities at two-tenths and eight-tenths the depth were independently measured 30 times at the central vertical line. All measured velocity profiles were approximately logarithmic distributions showed as examples (see Figure 5 as examples). From these profiles the values of Manning's n were computed and considered as true roughness values (the last column in Table 1).

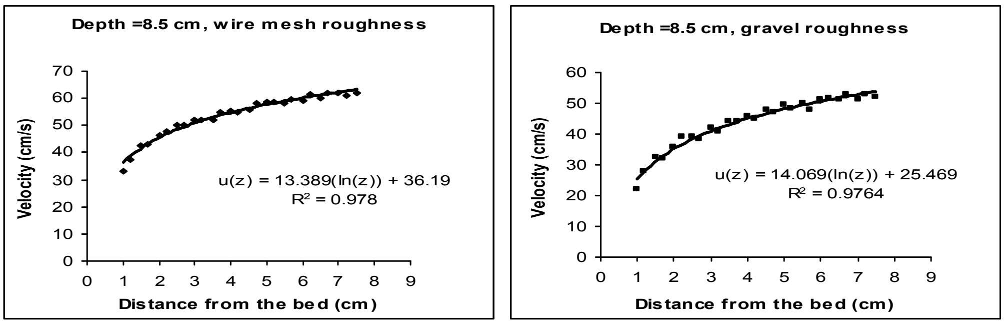


Figure 5. Measured velocity profiles

On the other hand, for each flow depth, 30 independent measured velocities were taken at two-tenths and eight-tenths the depth. From these measurements, 30 values of x and 30 values of n were computed using the two-point velocity formula (Equation 8). Then the relative errors in x and n were calculated as follows:

$$E_x = \frac{|x_i - \bar{x}|}{\bar{x}} 100\% \quad (17)$$

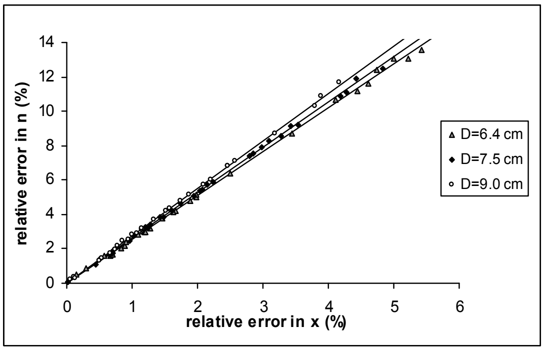
and

$$E_n = \frac{|n_i - n|}{n} 100\% \quad (18)$$

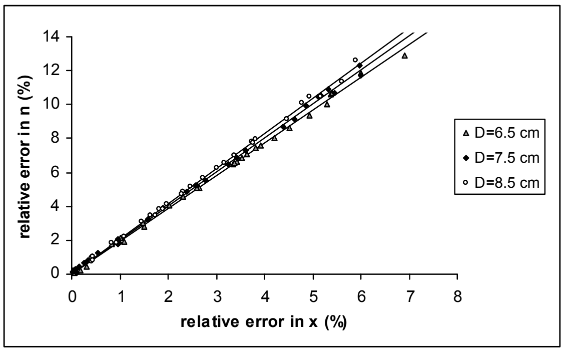
where xi is the ratio of $u_{0.2}$ and $u_{0.8}$ of i[th] measurement; n_i is the estimated Manning's n by using two-point velocity method of i[th] measurement; $\bar{x}$ is the mean value of x; n is the roughness coefficient computed from the whole velocity profile; E_x and E_n are the relative errors in xi and in n_i of i[th] measurement.

Figures 6 shows the relationships between relative errors in x and relative errors in n obtained from the experimental results for the wire mesh and the gravel bed respectively. From these figures, it can be seen that there is very good agreement between experimental results and the corresponding theoretical lines (Equation 16). This confirms that when using two-point velocity data to estimate the roughness coefficient, the greater the depth, the more sensitive the relative errors in estimated n are to the relative errors in x.

The relative errors in x were also plotted against the relative errors in n for the cases with the same depth ($D = 7.5$ cm) but with the two types of roughness (Figures 7). This figure shows clearly that the smoother a channel, the more sensitive the relative errors in n are to the relative errors in x. However, this figure also indicates that the rougher a channel, the higher the relative error in x, which results in a higher relative error in n. This finding is consistent with theoretical analysis.



(a)



(b)

Figure 6. Experimental relationships between relative errors in x and relative errors in estimated n and corresponding theoretical lines for (a) wire mesh (b) gravel bed

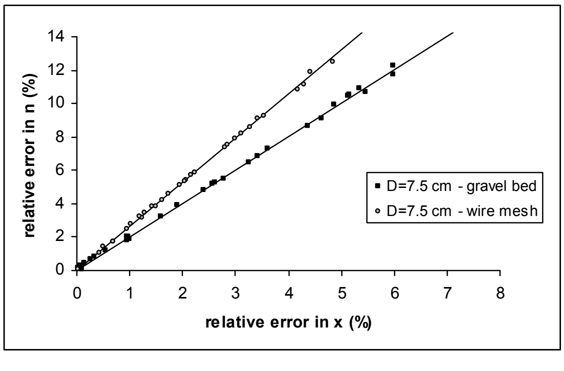


Figure 7. Experimental relationships between relative errors in x and relative errors in estimated n and corresponding theoretical lines for the same depth with of roughness types

5. CONCLUSIONS

In this paper, the two-point velocity method to estimate the roughness coefficient is re-investigate and a sensitive analysis is theoretically carried out and verified with experiment data. This study shows that that the relative error in n is more sensitive to the relative errors in x (the error of the ratio of velocity at two-tenths the depth to that at eight-tenths the depth) than in relative error in depth. The smoother and deeper a channel, the more sensitive the relative error in estimated n is to the relative error in x. However, for rougher channels with shallow depth, the errors in velocity measurement may be higher because of higher disturbance of roughness elements. Accordingly, the relative errors in x are also higher, which will result in higher relative errors in n. Therefore, this method should be used to estimate roughness coefficients with caution because measurement errors were unavoidable and/or the assumption of logarithm velocity distribution may have been violated.

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Tóm tắt:

PHÂN TÍCH ĐỘ NHẬY CỦA PHƯƠNG PHÁP XÁC ĐỊNH HỆ SỐ NHÁM SỬ DỤNG TÀI LIỆU ĐO LƯU TỐC

Việc xác định hệ số nhám Manning n có một ý nghĩa quan trọng trong tính toán thủy lực nói chung và thủy lực dòng hở nói riêng. Một trong những phương pháp đo đạc dòng chảy trong sông khá phổ biến là đo lưu tốc tại hai điểm ở 0.8 và 0.2 lần của độ sâu dòng chảy. Những số liệu này có thể áp dụng để xác định hệ số nhám dựa trên qui luật phân bố logarit của vận tốc trong dòng chảy rối. Bài báo này khảo sát lại phương pháp xác định hệ số nhám sử dụng số liệu đo lưu tốc và phân tích độ nhạy của kết quả tính toán bằng lý thuyết và thực nghiệm. Kết quả cho thấy có thể sử dụng số liệu đo lưu tốc để xác định hệ số nhám trong các sông rộng với chế độ chảy rối. Kết quả cũng chỉ ra rằng sai số tương đối của hệ số nhám rất nhạy với sai số tương đối của tỉ số lưu tốc hai điểm (x). Kết quả lý thuyết và thực nghiệm cho thấy, đối với các sông có độ nhám càng nhỏ và độ sâu càng lớn thì sai số tương đối của hệ số nhám tính toán càng nhạy với sai số tương đối của x.

Từ khóa: lòng dẫn hở, hệ số nhám, lưu tốc hai điểm, phân bố logarit.

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