VIBRATION OF FG SANDWICH BEAMS UNDER MOVING LOAD USING FIRST-ORDER SHEAR DEFORMABLE BEAM ELEMENT

Bui Van Tuyen[1]

Abstract: *The dynamic response of functionally graded (FG) sandwich beams excited by a moving point load is studied by the finite element method. Based on the first-order shear deformation beam theory, a finite beam element is formulated by using the hierachical shape functions. The beam is assumed to be formed from a homogeneous ceramic core and two symmetrical FG layers. The implicit Newmark method is employed in computing the dynamic response of the beams. The numerical results show that the formulated element is capable to access accurately the dynamic characteristics of the beam by using just several elements. A parametric study is carried out to highlight the effect of the material distribution, the core thickness to the beam height ratio and the moving load speed on the vibration characteristics.*

Keywords: FG sandwich beam, moving load, vibration, dynamic response, FEM.

1. INTRODUCTION

Analysis of beams subjected to moving loads is a classical problem in structural mechanics, and it has been a subject of investigation for a long time. This problem becomes a interesting topic in the field of structural mechanics since the date of invention of FG materials by Japanese scientists in 1984 (M. Akoizumi,1997). A combination of strong and light weight ceramics with traditional ductile metals remarkably enhances the vibration characteristics of the structures. Functionally graded (FG) sandwich material is a new type of composite which is widely used as structural material in recent years. This new composite has many advantages, including the high strength-to-weight ratio, good thermal resistance and no delaminating problem which often meet in the conventional composites. Investigations on the vibration analysis of FG sandwich beams have been extensively carried out recently.

The investigations on the dynamic response of FG beams (Simsek et al, 2009; 2010; Nguyen et al, 2013) in recent years have shown that the dynamic deflections of an FG metal-ceramic beam considerably reduces comparing to that of the pure beam. In addition, an FG beam induced by a soft core may improve the dynamic behavior of the structure when it subjected to moving loads. (Mohanty et al, 2012) proposed a finite element procedure for static and dynamic stability analysis of FG sandwich Timoshenko beams. (Bui et al, 2013) used the meshfree radial point interpolation method to study the vibration response of a cantilever FG sandwich beam subjected to a time-dependent tip load. Adopting the refined shear deformation theory, (Vo et al, 2014) investigated the free vibration and buckling of FG sandwich beams. In (Vo et al, 2015), presented a finite element model for the free vibration and buckling analyses of FG sandwich beams.

The present work aims to study the vibration of an FG sandwich beam excited by a moving harmonic load, which to the authors' best knowledge has not been investigated so far. The beam in this work is assumed to be formed from a homogeneous metallic soft core and two symmetrical FG skin layers. Based on the first-order shear deformation beam theory, a finite element beam formulation is derived and employed in computing the dynamic response of the beam. A parametric study is carried out to highlight the effect of the material distribution, the ratio of core thickness to beam height as well as the loading parameters on the vibration characteristics of the beam.

[1] *Facuty of Engineering, Thuyloi university, 175 Tay Son stress, Dong Da, Hanoi, Vietnam*

2. MATHEMATICAL FORMULATION

Figure 1 shows a simply supported FG sandwich beam with length *L*, height *h*, width *b*, core thickness h_c in a Cartesian co-ordinate system (*x*,*z*). The beam is assumed to be subjected to a point load *P*, moving from left to right at a constant speed *v*.

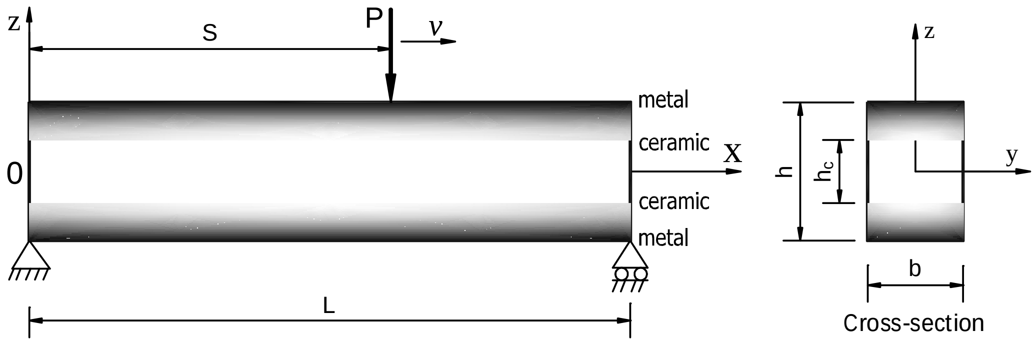

Figure 1. FG sandwich beam under a moving load

The beam is assumed to be formed from a ceramic soft core and two FG layers with the volume fraction of the constituent materials follows a power-law function as follows

$$\begin{cases} V_c^{(1)}(z) = \left(\dfrac{2z+h}{h-h_c}\right)^n & , \; z \in \left[-\dfrac{h}{2}, -\dfrac{h_c}{2}\right] \\ V_c^{(2)}(z) = 1 & , \; z \in \left[-\dfrac{h_c}{2}, \dfrac{h_c}{2}\right] \\ V_c^{(3)}(z) = \left(\dfrac{2z-h}{h_c-h}\right)^n & , \; z \in \left[\dfrac{h_c}{2}, \dfrac{h}{2}\right] \end{cases} \qquad (1)$$

and V_m=1-V_c. The subscripts '*c*' and '*m*' are used to indicate the 'ceramic' and 'metal', respectively. In Eq.(1), *n* is the material power-law index. From Eq.(1) one can see that the top and bottom surfaces of the beam are pure metal, and the core is full ceramic. The effective property *P*(*z*) (e.g., Young's modulus, shear modulus and mass density) can be evaluated by Voigt model.

Using the first-order shear deformation theory, the strain energy U_e and the kinetic energy T_e of the beam element are as follow (D.K. Nguyen and V.T. Bui , 2017)

$$U_e = \frac{1}{2}\int_0^l \int_A (\sigma_{xx}\varepsilon_{xx} + \tau_{xz}\gamma_{xz})dAdx = \frac{1}{2}\int_0^l \left[A_{11}u_{0,x}^2 - 2A_{12}u_{0,x}\theta_{,x} + A_{22}\theta_{,x}^2 + \psi A_{33}\left(w_{0,x} - \theta\right)^2 \right] dx \qquad (2)$$

and

$$\mathrm{T}_e = \frac{1}{2}\int_0^l \int_A \rho(z)\left(\dot{u}^2 + \dot{w}^2\right)dAdx = \frac{1}{2}\int_0^l \left[I_{11}\dot{u}_0^{\,2} + I_{11}\dot{w}_0^{\,2} - 2I_{12}\dot{u}_0\dot{\theta} + I_{22}\dot{\theta}^2 \right] dx \qquad (3)$$

in which A_{ij} and I_{ij} are the rigidities and mass moments of the beam element, respectively.

The finite element method is used. The beam is assumed being divided into a numbers of two-node beam elements with length of *l*. By using the hierachical shape functions, the shear strain γ_{xz} to constant (D.K. Nguyen and V.T. Bui , 2017), the vector of nodal displacements (**d)** for a generic beam element is given by

$$\mathbf{d} = \{u_1 \; w_1 \; \theta_1 \; \theta_3 \; u_2 \; w_2 \; \theta_2\}^T \qquad (4)$$

The displacements and rotation are interpolated from the nodal displacements

$$u_0 = \mathbf{N}_u\mathbf{d}, \;\; w_0 = \mathbf{N}_w\mathbf{d}, \;\; \theta = \mathbf{N}_\theta\mathbf{d} \qquad (5)$$

with

$$\mathbf{N}_u = \{N_1 \; 0 \;\; 0 \;\; 0 \; N_2 \; 0 \;\; 0\}^T, \; \mathbf{N}_\theta = \{0 \;\; 0 \;\; N_1 \; N_3 \; 0 \;\; 0 \;\; N_2\}^T,$$
$$\mathbf{N}_w = \left\{0 \;\; N_1 \;\; \frac{l}{8} N_3 \;\; \frac{l}{6} N_4 \;\; 0 \;\; N_2 \;\; -\frac{l}{8} N_3\right\}^T \tag{6}$$

in which N_1, N_2, N_3, N_4 are the hierachical shape functions (D.K. Nguyen and V.T. Bui , 2017).

One can write the strain energy (U), kinetic energy (T) in term of the nodal displacement vector as follows

$$U = \frac{1}{2}\sum_{i=1}^{n_{el}} \mathbf{d}^T \left(\mathbf{k}_{uu} + \mathbf{k}_{u\theta} + \mathbf{k}_{\theta\theta} + \mathbf{k}_{\gamma\gamma}\right)\mathbf{d}$$
$$T = \frac{1}{2}\sum_{i=1}^{n_{el}} \dot{\mathbf{d}}^T (\mathbf{m}_{uu} + \mathbf{m}_{ww} + \mathbf{m}_{u\theta} + \mathbf{m}_{\theta\theta})\dot{\mathbf{d}} \tag{7}$$

where n_{el} is the total number of the elements. The stiffness matrices $\mathbf{k}_{uu}$, $\mathbf{k}_{u\theta}$, $\mathbf{k}_{\theta\theta}$, $\mathbf{k}_{\gamma\gamma}$ and the mass matrices $\mathbf{m}_{uu}$, $\mathbf{m}_{ww}$, $\mathbf{m}_{u\theta}$, $\mathbf{m}_{\theta\theta}$ in (D.K. Nguyen and V.T. Bui, 2017).

Equations of motion for the beam in terms of finite element analysis as follows

$$\mathbf{M}\ddot{\mathbf{D}} + \mathbf{K}\mathbf{D} = \mathbf{F} \tag{8}$$

in which **M**, **K** and **F** respectively are the global mass, stiffness matrices and load vector. These matrices and vector are obtained by assembling the element mass, stiffness and load vector **m**, **k** and **f** derived above in the standard way of the finite element analysis. Eq. (8) can be solved by the direct integration Newmark method. Here, the average acceleration method which ensures the unconditional stability is employed.

3. NUMERICAL RESULTS

A simply supported beam composed of metal phase Alumina (Al_2O_3) core and FGM parts are composed of Aluminum and Alumina (Al and Al_2O_3). The properties of these component materials are Al: E_m=70 GPa, ρ_m=2702 kg/m^3, ν_m = 0.3, Al_2O_3: E_c=380 GPa, ρ_c =3960 kg/m^3, ν_c = 0.3. The slenderness ratio (L/h) of the beam is taken as L/h=20. The amplitude of the moving load is taken by P=100kN. For Newmark method in all the computations reported below, a uniform increment time step, t=T/500, T=L/v is the necessary total time for the load to cross the beam.

Table 1. Comparison of fundamental frequency parameter

		Ratio of hardcore						
n	Source	hc/h=0 (1-0-1)	hc/h=1/5 (2-1-2)	hc/h=1/4 (2-1-1)	hc/h=1/3 (1-1-1)	hc/h=2/5 (2-2-1)	hc/h=1 (1-2-1)	hc/h=4/5 (1-8-1)
0.5	Vo et al.,2014	4.3148	4.4290	4.4970	4.5324	4.6170	4.6979	5.1067
	Present work	4.3139	4.4281	4.3626	4.5316	4.4448	4.6973	5.1065
1	Vo et al.,2014	3.7147	3.8768	3.9774	4.0328	4.1602	4.2889	4.9233
	Present work	3.7137	3.8758	3.8305	4.0319	3.9884	4.2882	4.9231
2	Vo et al.,2014	3.1764	3.3465	3.4754	3.5389	3.7049	3.8769	4.7382
	Present work	3.1753	3.3455	3.3375	3.5379	3.5554	3.8761	4.7379

Table 1 lists the fundamental frequency parameter μ of the FG sandwich beam for various values of the core thickness to the beam height ratio h_C/h and the material index n. The frequency parameter in this Table is defined as follows: $\mu = \omega_1 L^2 / h\sqrt{\rho_m / E_m}$ in which ω_1 is the fundamental frequency of the beam, and ρ_m and E_m are the mass density and Young's modulus of the core material, respectively. Results of this paper are compared to the result obtained by using refined shear deformation theory in (Vo et al.,2014). As observed from the table, the present results are in good agreement with that of (Vo et al.,2014).

Table 2. Comparison of maximum dynamic deflection factor,max(fD), and moving load speed of FG beam

Source	Al_2O_3 (252 m/s*)	*n*=0.2 (222 m/s)	*n*=0.5 (198 m/s)	*n*=1 (179 m/s)	*n*=2 (164 m/s)
Present work	0.9382	1.0306	1.1509	1.2569	1.3450
Nguyen et al.,2017	0.9380	1.0402	1.1505	1.2566	1.3446
Khalili et al.,2010	0.9317	1.0233	1.1429	1.2486	1.3359

Note: *Moving load speed

In Table 2, the maximum dynamic deflection factor, max(fD), of the beam is given for values the material index $n.\mathrm{fD} = \max(w(L/2,t))/w_0$ where w_0 is the static deflection of homogeneous beam made of the pure material under a static load P_0 at the mid-span, $w_0 = P_0 L^3 / 48 E_m I$. As seen from Table 2, the maximum fD and the corresponding velocity of present work are in good agreement with that of (Nguyen et al, 2017; Khalili et al., 2010). It is worth to mention that the results in Tables 1 and 2 are converged by using twenty elements.

The effects of material index *n*, ratio hc/h and moving load speed on the dynamic response of the beam traversed by a single load are illustrated in Figure 2 and 3. Figure 2 shows relation between dynamic deflection factor and various values of index n with diffirent ratio hc/h at v=25m/s. With any value of the ratio h_c/h, when *n* increases, the dynamic deflection factor fD increases. And when h_c/h increases, the dynamic deflection factor fD decreases irrespective of the moving load speed. The relation between the moving load speed and the maximum mid-span deflection, as seen from Figure 3 is similar to that of the homogenous beam, and the largest deflect attains at a lower moving load speed for the beam associated with a higher index *n* and ratio lower h_c/h.

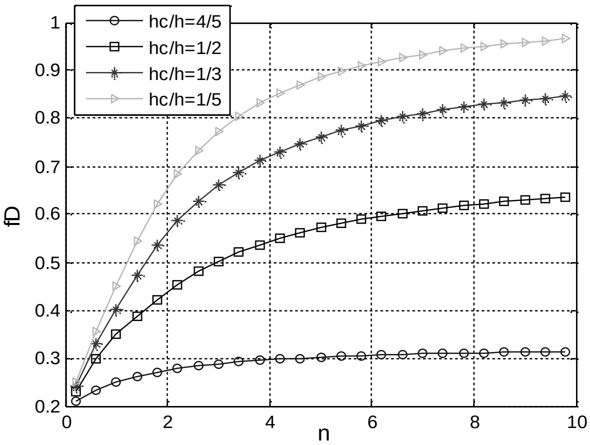

Figure 2. Relation between dynamic deflection factor and various values of index n with diffirent ratio hc/h : v=25m/s

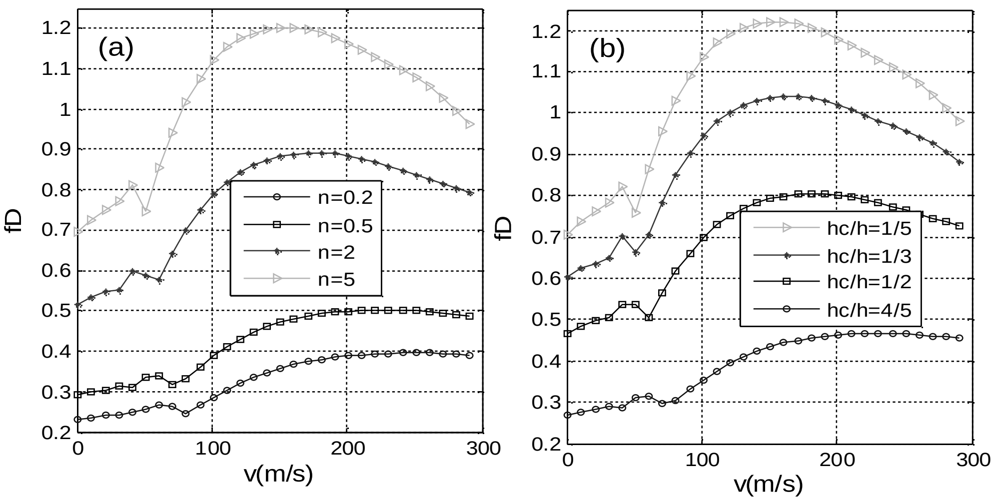

Figure 3. Relation between dynamic deflection factor with moving load speed for various indexes n: a) hc/h=1/3 and various index of n; b) n=3, various h_c/h.

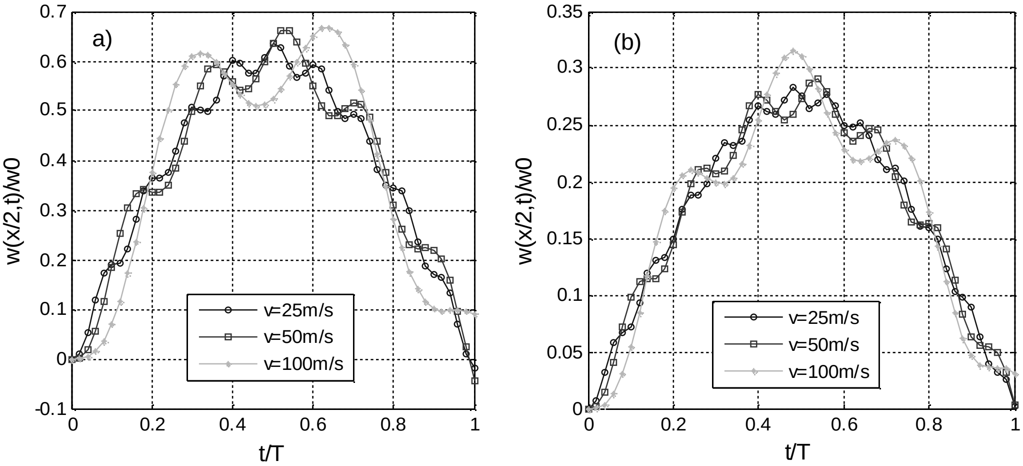

Figure 4. Time histories for normalized mid-span deformation with diffirent moving load speeds, n=3: a) hc/h=1/3; b) hc/h=4/5.

The effects of moving load speed on the time histories for mid-span deformation are illustrated in the figure 4. One can see that when moving load speed *v* increases, maximum deformation increases and oscillation frequency of the beam decreases. This trend also occurs for the higher h_c/h.

4. CONCLUSTION

The paper investigated the vibration of FG sandwich beam excited by a moving point load by using the finite element method. The beam is assumed to be formed from a homogeneous ceramic hard core and two symmetrical FG layer. A beam element based on the first-order shear deformation beam theory was formulated and employed in the investigation. The direct integration Newmark method has been used in computing the dynamic response of the beam. The numerical results have shown that the vibration characteristics of the beam, including the fundamental frequency and dynamic deflection factor, are strongly affected by the material distribution, the core thickness to the beam height ratio, the speed of the moving force. The dynamic deflection factor increases by increaising the index n and reducing the core thickness to beam height ratio. The moving speed not only alters the amplitude of the dynamic deflection but also changes the oscillation frequency of the beam.

REFERENCES

M. AKoizumi, FGM activities in Japan, Composites Part B: *Engineering*, 1997, 28: 1-4.

Mohanty, S.C., Dash R.R., & Rout, T., (2012) "*Static and dynamic stability analysis of a functionally graded Timoshenko beam*". International Journal of Structural Stability and Dynamics, Vol. 12(4), DOI: 10.1142/S0219455412500253.

Bui, T.Q., Khosravifard, A., Zhang, Ch., Hematiyan, M.R., & Golub, M.V.,(2013) *Dynamic analysis of sandwich beams with functionally graded core using a truly meshfree radial point interpolation method*, Engineering Structures, Vol. 47, p 90-104.

Vo, T.P., Thai, H.T., Nguyen, T.K., Maheri, A., & Lee, J., (2014) *Finite element model for vibration and buckling of functionally graded sandwich beams based on a refined shear deformation theory*, Engineering Structures, Vol. 64, p. 12-22.

Vo, T.P., Thai, H.T., Nguyen, T.K., Maheri, A., Inam, F., & Lee, J., , (2015) *A quasi-3D theory for vibration and buckling of functionally graded sandwich beams*, Composite Structures, Vol. 119, p. 1-12.

Şimşek, M., & Kocatürk, T.,(2009) *Free and forced vibration of a functionally graded beam subjected to a concentrated moving harmonic load*, Composite Structures, Vol. 90 (4), p. 465–473.

Şimşek, M.,(2010) *Vibration analysis of a functionally graded beam under a moving mass by using different beam theories*, Composite Structures, Vol. 92 (4), p. 904-917.

Nguyen, D.K., Gan, B.S., & Le, T.H.,(2013) *Dynamic response of non-uniform functionally graded beams subjected to a variable speed moving load*, Journal of Computational Science and Technology, JSME, Vol. 7(1), p. 12-27.

Le, T.H., Gan, B.S., Trinh, T.H., & Nguyen, D.K., (2014) *Finite element analysis of multi-span functionally graded beams under a moving harmonic load*. Mechanical Engineering Journal, Bulletin of the JSME, Vol. 1(3), p. 1-13.

D.K. Nguyen and V.T. Bui (2017), *Dynamic analysis of functionally graded Timoshenko beams in thermal environment using a higher-order hierarchical beam element. Mathematical Problems in Engineering.* DOI: https://doi.org/10.1155/2017/7025750.

Khalili SMR, Jafari AA, Eftekhari SA (2010) *A mixed Ritz-DQ method for forced vibration of functionally graded beams carrying moving loads*, Compos Struct 92: 2497-2511.

Tóm tắt:

DAO ĐỘNG CỦA DẦM SANDWICH FG DƯỚI TÁC ĐỘNG CỦA LỰC DI ĐỘNG SỬ DỤNG PHẦN TỬ BIẾN DẠNG TRƯỢT BẬC NHẤT

Đáp ứng động lực học của dầm sandwich FG chịu tác động của lực tập trung di động được nghiên cứu bằng phương pháp phần tử hữu hạn. Phần tử dầm được dùng để tính toán sử dụng hàm dạng thứ bậc dựa trên lý thuyết biến dạng trượt bậc nhất. Dầm được cấu trúc từ lõi gốm và 2 lớp vật liệu có cơ tính biến thiên đối xứng. Phương pháp tích phân trực tiếp Newmark ẩn được sử dụng để tính toán đáp ứng động lực học của dầm. Kết quả số cho thấy với một số phần tử được thành lập có khả năng đáp ứng tốt đến bức tranh dao động của dầm. Nghiên cứu cũng làm sáng tỏ ảnh hưởng của sự phân bố tham số vật liệu, tỷ lệ chiều cao lớp lõi và tốc độ của lực di động đến đặc trưng dao động của dầm.

Từ khóa: dầm sandwich FG, lực di động, dao động, đáp ứng động lực học, FEM

Ngày nhận bài: 17/7/2019

Ngày chấp nhận đăng: 28/8/2019