Nonlinear buckling analysis of sandwich functionally graded plates with grid core resting on elastic foundation

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ABSTRACT: Based on Kirchhoff's plate theory with the geometrical nonlinearity in von Karman sense and the smeared stiffeners technique, the governing equations of sandwich functionally graded plates with grid core resting on Pasternak's elastic foundation are derived in this paper. The nonlinear buckling behavior of sandwich plate acted on by axial loading is considered with the effects of elastic foundation by using the Galerkin method. The critical buckling loads are found according to the bifurcation criterion. Some numerical results are given and some significant remarks are obtained.

KEYWORDS: Functionally graded material, sandwich structure, buckling, plate, grid core.

1. INTRODUCTION

Functionally graded material (FGM) has wide applications in many engineering designs. In the last decade, many works focused on the linear and nonlinear mechanical behavior of stiffened and unstiffened FGM plates. Linear and nonlinear bending and buckling analysis of FGM plate subjected to various load types with and without the temperature effects was performed in many works by using the different plate theories and solution method [1-3].

An improved Lekhnitskii's smeared stiffener technique for eccentrically oblique stiffener system was proposed by Nam et al. [5,6]. This technique was applied to the nonlinear buckling problems of FGM plates reinforced by eccentrically oblique stiffeners subjected to thermal and mechanical loads. The obtained results showed that the effects of oblique stiffeners on the buckling behavior of plates are significant. Recently, Khoa et al. [7] have studied the nonlinear buckling analysis of FGM plates and cylindrical panels with grid core, without elastic foundation. The grid core is only designed by an oblique stiffener system.

This paper presents the new design for grid core by combining the oblique, transversal, and longitudinal stiffener systems. The equilibrium equation system of nonlinear buckling of sandwich plates with symmetric functionally graded face sheets resting on Pasternak elastic foundation and subjected to mechanic loads is established by using the Kirchhoff's plate theory with von Karman nonlinearities of deflection and the improved the Lekhnitskii's smeared stiffener technique are established in this paper.

2. SANDWICH FGM PLATES WITH GRID CORE AND SOLUTION PROBLEM

Consider a sandwich plate with FGM face sheets and grid core of total thickness h and in-plane edges a and b as shown in *Figure 2.1*. The plate is rested on the Pasternak's elastic foundation with K_1, K_2 parameters. The grid core is designed by longitudinal, transversal and oblique stiffeners *(Figure 2.1)* and made by isotropic material. The elastic modulus of plate can be written by according to:

For upper FGM layer

$$E(z)=E_{ex}+(E_{in}-E_{ex})\left(\frac{-2z+h}{h-2h_{in}}\right)^k, h_{in}\le z\le\frac{h}{2}, \tag{1}$$

for lower FGM layer

$$E(z)=E_{ex}+(E_{in}-E_{ex})\left(\frac{2z+h}{h-2h_{in}}\right)^k, -\frac{h}{2}\le z\le -h_{in}, \tag{2}$$

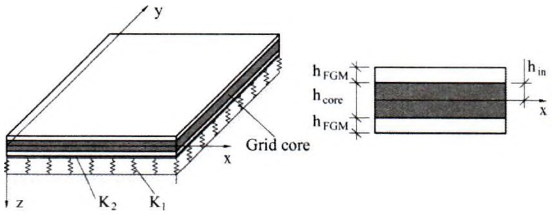
and for Grid Core

where: E_{ex}, E_{in} are the elastic moduli of external surface and internal surface of FGM face sheets, respectively.

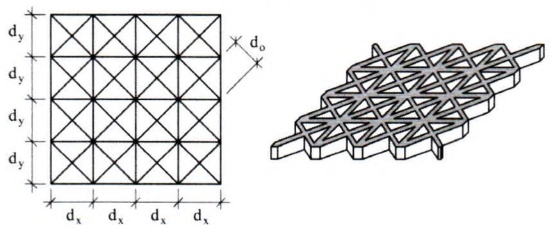
The Poisson's ratio is assumed to be constant.

According to the Kirchhoff's plate theory and geometrical nonlinearity in von Karman sense, the strains at the middle surface and curvatures are related to the displacement components u, v, w in the x, y, z coordinate directions as:

$$\begin{aligned}
&\varepsilon_x^0=\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2, \quad \chi_x=\frac{\partial^2 w}{\partial x^2},\\
&\varepsilon_y^0=\frac{\partial v}{\partial y}+\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2, \quad \chi_y=\frac{\partial^2 w}{\partial y^2},\\
&\gamma_{xy}^0=\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}+\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}, \quad \chi_{xy}=\frac{\partial^2 w}{\partial x\partial y},
\end{aligned} \tag{3}$$



Configuration of sandwich plate



Configuration of Grid core

Figure 2.1: Configuration of a sandwich functionally graded plates with grid core resting on elastic foundation

The strains across the plate thickness at a distance Z from the mid-surface are:

$$\varepsilon_x = \varepsilon_x^0 - z\chi_x, \quad \varepsilon_y = \varepsilon_y^0 - z\chi_y, \quad \gamma_{xy} = \gamma_{xy}^0 - 2z\chi_{xy}, \tag{4}$$

The deformation compatibility equation can be obtained from Eq. (3), as:

$$\frac{\partial^2 \varepsilon_x^0}{\partial y^2} + \frac{\partial^2 \varepsilon_y^0}{\partial x^2} - \frac{\partial^2 \gamma_{xy}^0}{\partial x \partial y} = \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} \tag{5}$$

By using Hookian stress-strain relationship of plate and taking into account the contribution of grid core by the smeared stiffeners technique and integrating the stress - strain expressions and their moments through the thickness of plate, the expressions for force and moment resultants of a sandwich FGM plate with grid core is obtained as:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & 0 & 0 & 0 \\ A_{12} & A_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & A_{66} & 0 & 0 & 0 \\ 0 & 0 & 0 & D_{11} & D_{12} & 0 \\ 0 & 0 & 0 & D_{12} & D_{22} & 0 \\ 0 & 0 & 0 & 0 & 0 & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \chi_x \\ \chi_y \\ 2\chi_{xy} \end{Bmatrix}, \tag{6}$$

where:

$$A_{11} = \frac{E_1}{1-\nu^2} + \frac{E_{in}A_x}{d_x} + 2\frac{E_{in}A_o}{d_o}\cos^4\theta, \qquad A_{22} = \frac{E_1}{1-\nu^2} + \frac{E_{in}A_y}{d_y} + 2\frac{E_{in}A_o}{d_o}\cos^4\theta,$$

$$A_{12} = \frac{E_1\nu}{1-\nu^2} + 2\frac{E_{in}A_o}{d_o}\sin^2\theta\cos^2\theta, \qquad A_{66} = \frac{E_1}{2(1+\nu)} + 2\frac{E_{in}A_o}{d_o}\sin^2\theta\cos^2\theta,$$

$$D_{11} = D_{22} = \frac{E_3}{(1-\nu^2)} + \frac{E_{in}I_x}{d_x} + 2\frac{E_{in}I_o}{d_o}\cos^4\theta, \quad D_{22} = \frac{E_3}{(1-\nu^2)} + \frac{E_{in}I_y}{d_y} + 2\frac{E_{in}I_o}{d_o}\cos^4\theta,$$

$$D_{12} = \frac{E_3\nu}{(1-\nu^2)} + 2\frac{E_{in}I_o}{d_o}\sin^2\theta\cos^2\theta, \qquad D_{66} = \frac{E_3}{2(1+\nu)} + 2\frac{E_{in}I_o}{d_o}\sin^2\theta\cos^2\theta,$$

$$E_1 = E_{ou}(h-2h_{in}) + (E_{in}-E_{ex})\frac{(h-2h_{in})}{k+1},$$

$$E_3 = E_{ex}\frac{h^3}{12} - 2E_{ex}\frac{h_{in}^3}{3} + (E_{in}-E_{ex})\left[\frac{(h-2h_{in})^3}{4(k+3)} - \frac{(h-2h_{in})^2 h}{2(k+2)} + \frac{(h-2h_{in})h^2}{4(k+1)}\right].$$

The nonlinear equilibrium equations of a sandwich FGM plate with grid core based on the Kirchhoff's plate theory are given by:

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0, \quad \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = 0,$$

$$\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} + \tag{7}$$

$$+ N_x\frac{\partial^2 w}{\partial x^2} + 2N_{xy}\frac{\partial^2 w}{\partial x \partial y} + N_y\frac{\partial^2 w}{\partial y^2} - K_1 w + K_2\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) = 0,$$

The first two of Eq.(7) are satisfied automatically by choosing a stress function φ as:

$$N_x = \frac{\partial^2 \varphi}{\partial y^2}, \quad N_y = \frac{\partial^2 \varphi}{\partial x^2}, \quad N_{xy} = -\frac{\partial^2 \varphi}{\partial x \partial y}. \tag{8}$$

The compatibility and equilibrium equations can be rewritten by in the forms, as:

$$A_{11}^*\frac{\partial^4 \varphi}{\partial x^4} + \left(A_{66}^* - 2A_{12}^*\right)\frac{\partial^4 \varphi}{\partial x^2 \partial y^2} + A_{22}^*\frac{\partial^4 \varphi}{\partial y^4} + B_{21}^*\frac{\partial^4 w}{\partial x^4} +$$
$$+\left(B_{11}^* + B_{22}^* - 2B_{66}^*\right)\frac{\partial^4 w}{\partial x^2 \partial y^2} + B_{12}^*\frac{\partial^4 w}{\partial y^4} = \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}, \tag{9}$$

$$D_{11}^*\frac{\partial^4 w}{\partial x^4} + \left(D_{12}^* + D_{21}^* + 4D_{66}^*\right)\frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{22}^*\frac{\partial^4 w}{\partial y^4} - B_{21}^*\frac{\partial^4 \varphi}{\partial x^4} -$$
$$-\left(B_{11}^* + B_{22}^* - 2B_{66}^*\right)\frac{\partial^4 \varphi}{\partial x^2 \partial y^2} - B_{12}^*\frac{\partial^4 \varphi}{\partial y^4}$$
$$-\frac{\partial^2 \varphi}{\partial y^2}\frac{\partial^2 w}{\partial x^2} + 2\frac{\partial^2 \varphi}{\partial x \partial y}\frac{\partial^2 w}{\partial x \partial y} - \frac{\partial^2 \varphi}{\partial x^2}\frac{\partial^2 w}{\partial y^2} + K_1 w - K_2\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) = 0, \tag{10}$$

Supposing that a sandwich FGM plate with grid core is simply supported and subjected to in plane compressive load of intensities r at its cross-section. The considered condition can be satisfied identically if the buckling mode shape is represented by:

$$w = f\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \tag{11}$$

Substituting Eq. (11) into Eq. (10) and solving obtained equation to determine the form of stress function, then, substituting of expressions of stress function and solution of deflection into Eq. (10) and applying the Galerkin method to the resulting equation yield.

$$\bar{D}f + \left(\frac{8mn\lambda^2}{3\pi^2}\frac{B}{A}\delta_1\delta_2 + H\right)f^2 + Kf^3 - \frac{a^2h}{\pi^2}rm^2 f = 0 \tag{12}$$

The obtained equation (12) is the governing equation for nonlinear buckling analysis of sandwich FGM plates with grid core resting on elastic foundation in general. Based on this equation, the nonlinear buckling behavior of plates can be investigated in the next section.

3. NUMERICAL INVESTIGATIONS

To evaluate the effects of Grid Core on the nonlinear buckling behavior of studied structures, the plates are considered with a = 1 m, b = 1 m. The total thickness of plate is h = 0.01m. The height of stiffeners (core thickness) is $h_{core} = 2h_{in} = 0.006$ m, the width of stiffeners is $b_x = b_y = b_o = 0.002$ m and the number of stiffeners $n_x = n_y = 10$, the angle of oblique stiffeners is chosen as θ = π/4. The combination of materials consists of Aluminum (metal) and Alumina (Ceramic). The plate is considered with ceramic is in external surface and metal is in internal surface and grid core is made by full metal. Therefore $E_{ex} = E_{ceramic} = 380$ GPa and $E_{in} = E_{metal = 70}$ GPa. The Poisson's ratio v is chosen to be 0.3 for simplicity.

Table 3.1. Critical buckling loads of FGM plates with grid core (without elastic foundation) under axial compression (GPa)

	k=0.2	k=1	k=10
h_{core}=0.004	0.4787	0.9031	1.2489
h_{core}=0.006	0.3800	0.7114	1.0298
h_{core}=0.008	0.2284	0.4216	0.6341

The critical buckling loads of sandwich FGM plates with grid core under axial compression are shown in *Table 3.1*. The results show that the volume fraction index k largely affects the critical buckling of the plate. The critical buckling load increases when the volume fraction index k increases. The obtained results also depend on the thickness of the grid core. The critical buckling load decreases when the thickness of the grid core increases.

The postbuckling curves of sandwich FGM plates with grid core are presented in *Figs. 3.1 - 3.4*. Clearly, the bifurcation phenomenon can be observed in these figures, and the bifurcation point is corresponding with the critical buckling of sandwich plates.

Effect of volume fraction index on the postbuckling curves of grid core FGM plates is presented in *Figure 3.1*. As can be seen that the postbuckling strength is larger with the larger volume fraction index.

Similarly, the effect of core thickness on the postbuckling curves of grid core FGM plates can be seen in *Figure 3.2*. As can be observed, the postbuckling strength of plates is larger with the thinner core layer.

Figs. 3.3 - 3.4 present the postbuckling curves of grid core FGM plates with different Winkler and Pasternak parameters, respectively. The results show the foundation parameters have a positive effect on the postbuckling behavior of the plate.

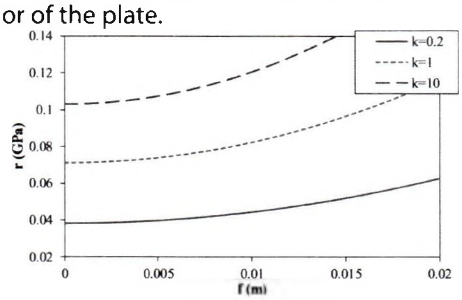


Figure 3.1: Postbuckling of grid core FGM plates (without elastic foundation) with different volume fraction indexes (h_{core} = 0.006 m)

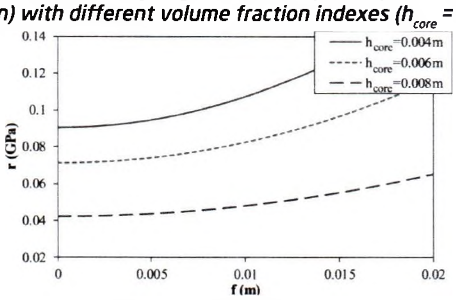


Figure 3.2: Postbuckling of grid core FGM plates (without elastic foundation) with different core thicknesses (k = 1)

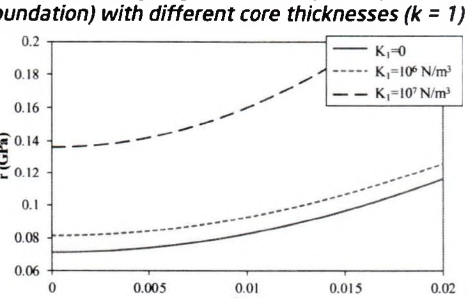


Figure 3.3: Postbuckling of grid core FGM plates with different Winkler parameters (K_2 = 0, k = 1, h_{core} = 0.006 m)

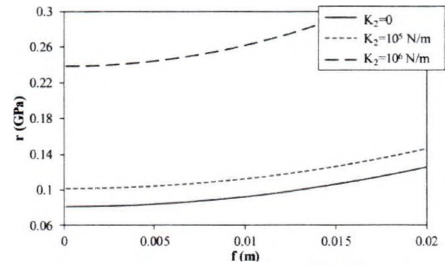


Figure 3.4: Postbuckling of grid core FGM plates with different Pasternak parameters (K_1 = 10^6 N/m^3, k = 1, h_{core} = 0.006 m)

4. CONCLUSION

An analytical approach of nonlinear buckling and postbuckling of Grid Core Sandwich plates with symmetric functionally graded face sheets resting on Pasternak foundation subjected to mechanic load based on the Kirchhoff's plate theory and the homogenization technique with von Kármán nonlinearities is presented in this paper. The nonlinear equation system of plates is established by using the Galerkin method. The critical buckling and postbuckling behavior of plates are numerically investigated. Effects of grid core, volume fraction index of FGM, foundation parameters on the nonlinear buckling behavior of plates are investigated and evaluated.

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