TRACKING CONTROL OF WHEELED MOBILE ROBOT WITH MODEL UNCERTAINTIES AND INPUT DISTURBANCE: A NOVEL APPROACH WITH DISTURBANCE ESTIMATION AND ARBITRARY CONVERGENCE TIME CONTROLLER

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ABSTRACT

Wheeled mobile robots have been widely applied in practice and they have drawn a lot of interests from the research community due to their non-holonomic constraints, nonlinearity and uncertain load. In this work, a novel tracking control approach is proposed for wheeled mobile robots under model uncertainties and input disturbance. The new approach is based on a disturbance estimator and an arbitrary convergence time controller. The model uncertainties and input disturbance will be compensated by the disturbance estimator whereas velocity errors will converges to zero in small prescribed settling time by the arbitrary convergence time controller, which will improve control performance of the closed-loop system. The effective of the proposed method will be verified through numerical simulations.

ĐIỀU KHIỂN BÁM CHO XE TỰ HÀNH VỚI MÔ HÌNH BẤT ĐỊNH VÀ NHIỄU ĐẦU VÀO: MỘT PHƯƠNG PHÁP MỚI VỚI BỘ QUAN SÁT VÀ BỘ ĐIỀU KHIỂN THỜI GIAN HỮU HẠN TÙY Ý

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TỪ KHÓA

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Thời gian hội tụ tùy ý
Xe tự hành
Điều khiển phi tuyến
Điều khiển bám

TÓM TẮT

Xe tự hành đã được sử dụng rộng rãi trong thực tế và chúng thu hút được nhiều sự quan tâm từ những nhà nghiên cứu do tính ràng buộc không tích phân được, tính phi tuyến và tải bất định của chúng. Trong bài báo này một phương pháp điều khiển bám mới được đề xuất cho xe tự hành với mô hình bất định và có nhiễu đầu vào. Phương pháp mới này dựa trên một bộ quan sát nhiễu đầu vào và bộ điều khiển có thời gian đáp ứng tùy ý. Bất định của mô hình và nhiễu đầu vào sẽ được bù bằng bộ quan sát nhiễu trong khi đó sai lệch tốc độ sẽ tiến đến không trong một khoảng thời gian xác lập cho trước bởi bộ điều khiển thời gian hữu hạn tùy ý, bộ điều khiển này sẽ cải thiện chất lượng điều khiển của hệ kín. Tính hiệu quả của phương pháp được kiểm chứng thông qua mô phỏng số.

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1. Introduction

Automated Guided Vehicles (AGVs) have been widely studied and applied in industry [1]. There have several types of AGVs but the type of three-wheeled mobile robot (WMR) [2] will be considered in this work due to its nonlinearity, underactuation property and nonholonomic constraint, which leads to on of the most difficult control problems for AGVs.

Some advanced control methods have been developed for the WMR such as adaptive sliding mode control [3], nonlinear control based on extended state observer [4], adaptive tracking control [5] for the WMR with the centre located at the middle of wheels' axis and model predictive control [6]. These controllers were either complex in implementation or require high computational load.

In this paper, a novel supplemental method to the existing ones is proposed for tracking control of WMR under conditions of model uncertainty and input disturbance. The main contribution of this work is a) to develop a finite time controller for the velocity of WMR and b) to apply a novel disturbance estimatimation technique for removing effect of uncertainty and disturbance.

The remaining part of this paper is organized as follows. In section 2, a mathematical model of the WMR is briefly given first, then a traditional tracking controller is provided, after that an arbitrary time convergence controller is designed to improve control performance by producing desired velocities for tracking controller as fast as possible, finally a disturbance estimator is developed to supress the impact of model uncertainty and disturbance. In sec tion 3, numerical simulations are carried out for the WMR and comparison is also made when the disturbance estimator is not applied. Final section will draw some conclusions and provide future work.

2. Main results

2.1. Mathematical model

A schematic diagram of WMRs is shown in Fig. 1, in which (x_c, y_c) are center coordinates of the WMR, d is the distance between the center and wheel's axis, r is the radius of wheels, $2R$ is the distance between two wheels and θ is the WMR's orientation angle. A model of the WMR [2] can be represented as:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{V}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{F}(\dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q}) + \boldsymbol{\tau}_{\mathrm{d}} = \mathbf{B}(\mathbf{q})\boldsymbol{\tau} - \mathbf{A}^{\mathrm{T}}(\mathbf{q})\lambda, \tag{1}$$

where $\mathbf{q} = [x_c, y_c, \theta]^T$, $\mathbf{M}(\mathbf{q}) \in \mathbb{R}^{3\times 3}$ is a symmetric, positive definite inertia matrix, $\mathbf{V}(\mathbf{q}, \dot{\mathbf{q}}) \in \mathbb{R}^{3\times 3}$ is the centripetal and Coriolis matrix, $\mathbf{F}(\dot{\mathbf{q}}) \in \mathbb{R}^{3\times 1}$ is the surface friction vector, $\mathbf{G}(\mathbf{q}) \in \mathbb{R}^{3\times 1}$ is the gravitational vector, $\boldsymbol{\tau}_{\mathrm{d}}$ is the unknown disturbance vector, $\mathbf{B}(\mathbf{q}) \in \mathbb{R}^{3\times 2}$ is the input transformation matrix, $\mathbf{A}(\mathbf{q}) \in \mathbb{R}^{1\times 3}$ is the matrix associated with constraints and $\lambda \in \mathbb{R}^{1\times 1}$ is the constraint force vector.

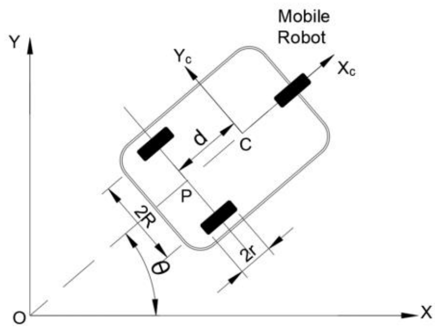

Figure 1. *A schematic diagram of WMR.*

It is assumed that the WMR can only roll, and it does not slip [2]. So, the following equation holds:

$$\dot{y}_c cos\theta - \dot{x}_c sin\theta - d\dot{\theta} = 0. \tag{2}$$

Let

$$\mathbf{S}(\mathbf{q}) = \begin{bmatrix} cos\theta & -dsin\theta \\ sin\theta & dcos\theta \\ 0 & 1 \end{bmatrix} \text{ and } \mathbf{v} = \begin{bmatrix} v \\ \omega \end{bmatrix},$$

where v and ω are linear and angular velocities, respectively. Then,

$$\begin{bmatrix} \dot{x}_c \\ \dot{y}_c \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} cos\theta & -dsin\theta \\ sin\theta & dcos\theta \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}. \tag{3}$$

The following terms are obtained by using the Lagrange method, in which $\boldsymbol{G(q)} = \mathbf{0}$ due to assumption of moving on horizontal plane.

$$\mathbf{M}(\mathbf{q}) = \begin{bmatrix} m & 0 & mdsin\theta \\ 0 & m & -mdcos\theta \\ mdsin\theta & -mdcos\theta & I \end{bmatrix}, \quad \mathbf{V}(\mathbf{q}, \dot{\mathbf{q}}) = \begin{bmatrix} 0 & 0 & md\dot{\theta}\cos\theta \\ 0 & 0 & md\dot{\theta}\sin\theta \\ 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{A}(\mathbf{q})^{\mathrm{T}} = \begin{bmatrix} -sin\theta \\ cos\theta \\ -d \end{bmatrix}, \quad \mathbf{B}(\mathbf{q}) = \frac{1}{r} \begin{bmatrix} cos\theta & cos\theta \\ sin\theta & sin\theta \\ R & -R \end{bmatrix}$$

$$\lambda = -m(\dot{x}_c cos\theta + \dot{y}_c \sin\theta)\dot{\theta}$$

Thus,

$$\mathbf{S}^{\mathrm{T}}(\mathbf{q})\mathbf{A}^{\mathrm{T}}(\mathbf{q}) = 0 \tag{4}$$

Premultiplying both sides of Eq. (1) with $\mathbf{S}^{\mathrm{T}}$ to have

$$\dot{\mathbf{q}} = \mathbf{S}\mathbf{v} \tag{5}$$

$$\mathbf{S}^{\mathrm{T}}\mathbf{M}\mathbf{S}\dot{\mathbf{v}} + \mathbf{S}^{\mathrm{T}}\left(\mathbf{M}\dot{\mathbf{S}} + \mathbf{V}\mathbf{S}\right)\mathbf{v} + \mathbf{S}^{\mathrm{T}}\mathbf{F} + \mathbf{S}^{\mathrm{T}}\boldsymbol{\tau}_{\mathbf{d}} = \mathbf{S}^{\mathrm{T}}\mathbf{B}\boldsymbol{\tau} \tag{6}$$

Denote $\overline{\mathbf{M}}(\mathbf{q}) = \mathbf{S}^{\mathrm{T}}\mathbf{M}\mathbf{S}, \overline{\mathbf{V}}(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{S}^{\mathrm{T}}\left(\mathbf{M}\dot{\mathbf{S}} + \mathbf{V}\mathbf{S}\right), \overline{\mathbf{F}}(\mathbf{v}) = \mathbf{S}^{\mathrm{T}}\mathbf{F}, \overline{\boldsymbol{\tau}}_{\mathbf{d}} = \mathbf{S}^{\mathrm{T}}\boldsymbol{\tau}_{\mathbf{d}}, \overline{\mathbf{B}} = \mathbf{S}^{\mathrm{T}}\mathbf{B}$. Then, Eq. (6) is rewritten as:

$$\overline{\mathbf{M}}(\mathbf{q})\dot{\mathbf{v}} + \overline{\mathbf{V}}(\mathbf{q}, \dot{\mathbf{q}})\mathbf{v} + \overline{\mathbf{F}}(\mathbf{v}) + \overline{\boldsymbol{\tau}}_{\mathbf{d}} = \overline{\mathbf{B}}\boldsymbol{\tau} \tag{7}$$

The system (7) will be used to design controllers in next section.

2.2. Tracking control

Given reference signal $\mathbf{q}_{\mathbf{r}}(t) = [x_r(t), y_r(t), \theta_r(t)]^T$. The target is to find forces $\boldsymbol{\tau}(t)$ applying to left and right wheels to the WMR such that $\mathbf{q}(t) - \mathbf{q}_{\mathbf{r}}(t) \to 0$ as $t \to \infty$. This control problem poses some following issues: 1) the system is under-actuated because there are two inputs τ_1, τ_2 but three output $x(t), y(t), \theta(t)$; 2) it is affected by disturbance and model uncertainty; 3) there exists a non-holonomic constraint; and 4) the references are time varying.

A block diagram of the proposed control method is shown in Fig. 2.

The proposed method consists of two control loops where the inner loop including an arbitrary convergence time controller [7] and disturbance estimator [8], and the outer loop is a traditional controller [3].

Let $\mathbf{v}_{\boldsymbol{r}}(t) = [v_r(t) \quad \omega_r(t)]^T$ be the reference velocity. According to Eq. (5), it is obtained:

$$\dot{\mathbf{q}}_{\mathbf{r}} = \mathbf{S}\mathbf{v}_{\mathbf{r}} \tag{8}$$

Define

$$\mathbf{q}_{\mathbf{e}} = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} cos\theta & sin\theta & 0 \\ -sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x \\ y_r - y \\ \theta_r - \theta \end{bmatrix}. \tag{9}$$

Then,

$$\dot{\mathbf{q}}_{\mathbf{e}} = \begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{bmatrix} = v \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} + \omega \begin{bmatrix} e_2 \\ -e_1 \\ -1 \end{bmatrix} + \begin{bmatrix} v_r cose_3 \\ v_r sine_3 \\ \omega_r \end{bmatrix} \tag{10}$$

To achieve that $\mathbf{q}(t) - \mathbf{q}_{\mathbf{r}}(t) \to \mathbf{0}$ as $t \to \infty$, the outer loop controller [3] is designed as:

$$\begin{bmatrix} v_c \\ \omega_c \end{bmatrix} = \begin{bmatrix} v_r cose_3 + k_1 e_1 \\ \omega_r + k_2 v_r e_2 + k_3 v_r sine_3 \end{bmatrix}, \tag{11}$$

where k_1, k_2, k_3 are positive constants. The overall control system is shown in Fig. 2.

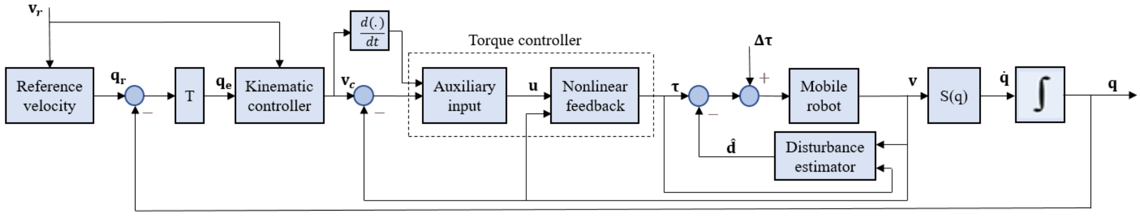

Figure 2. *Control system diagram.*

2.3. Velocity controller design

To force the WMR's velocities to track the output of the kinematic controller (v_c, ω_c) as soon as possible, we will design an arbitrary convergence time controller using the novel control technique [7].

It is assumed that all the uncertainties and disturbances are zero. These unknown terms will be compensated by their estimated values from a disturbance estimator. Define $\mathbf{u} = [u_1 \quad u_2]^T$ as new auxiliary input and design a feedback linearization controller as

$$\boldsymbol{\tau} = f(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{v}, \mathbf{u}) = \overline{\mathbf{B}}^{-1}(\mathbf{q})[\overline{\mathbf{M}}(\mathbf{q})\mathbf{u} + \overline{\mathbf{V}}(\mathbf{q}, \dot{\mathbf{q}})\mathbf{v}]. \tag{12}$$

Then, the system (7) becomes

$$\dot{\mathbf{v}} = \mathbf{u} \tag{13}$$

Denote velocity error vector as $\mathbf{e_v} = \begin{bmatrix} e_v \\ e_\omega \end{bmatrix} = \mathbf{v_c} - \mathbf{v} = \begin{bmatrix} v_c - v \\ \omega_c - \omega \end{bmatrix}$.

Theorem 1.

With following control law

$$\mathbf{u} = \mathbf{C} + \dot{\mathbf{v}}_\mathbf{c}\,, \tag{14}$$

where

$$\mathbf{C} = \begin{cases} \begin{bmatrix} \dfrac{-\eta_1(e^{-e_v} - 1)}{e^{-e_v}(t_f - t)} \\ \dfrac{-\eta_2(e^{-e_\omega} - 1)}{e^{-e_\omega}(t_f - t)} \end{bmatrix} & \text{for } t < t_f \\ \mathbf{0}, & \text{for } t \geq tf \end{cases} \tag{15}$$

$\mathbf{e_v} \to \mathbf{0}$ after an arbitrary small time $t_f > 0$.

Proof:

Choose a Lyapunov candidate function as

$$V_d = \|\mathbf{e_v}\|^2 \tag{16}$$

Clearly, $V_d \geq 0$ and $V_d = 0$ if and only if $\mathbf{e_v} = \mathbf{0}$. Time derivative of V_d is

$$\dot{V}_d = 2\mathbf{e_v}^T\dot{\mathbf{e}}_\mathbf{v} = 2\mathbf{e_v}^{\mathrm{T}}(\dot{\mathbf{v}}_\mathbf{c} - \dot{\mathbf{v}}) = 2\mathbf{e_v}^{\mathrm{T}}(\dot{\mathbf{v}}_\mathbf{c} - \mathbf{u}) = -2\mathbf{e_v}^{\mathrm{T}}\mathbf{C} \tag{17}$$

Consider a function $f_1(x) = \frac{x(e^{-x}-1)}{e^{-x}} = x(1 - e^x)$. One has

$$f_1(x) - f_1(-|x|) = \begin{cases} x(1 - e^x) + x(1 - e^{-x}) = x(2 - e^x - e^{-x}), \text{when } x \geq 0 \\ 0 \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad , \text{when } x < 0 \end{cases} \tag{18}$$

We have $2 - e^x - e^{-x} \leq 0, \forall x$, but $x > 0 \Rightarrow f_1(x) - f_1(|x|) < 0$. Thus,

$$f_1(x) \leq f_1(-|x|), \forall x \tag{19}$$

(i) As $t < t_f$

$$\dot{V}_d = -2\left[-\frac{\eta_1 e_v(e^{-e_v}-1)}{e^{-e_v}(t_f-t)} - \frac{\eta_2 e_\omega(e^{-e_\omega}-1)}{e^{-e_\omega}(t_f-t)}\right] = 2\left[\frac{\eta_1 e_v(e^{-e_v}-1)}{e^{-e_v}(t_f-t)} + \frac{\eta_2 e_\omega(e^{-e_\omega}-1)}{e^{-e_\omega}(t_f-t)}\right] \quad (20)$$

From (19), one gets:

$$\dot{V}_d \le -2\left[\frac{\eta_1|e_v|(e^{|e_v|}-1)}{e^{|e_v|}(t_f-t)} + \frac{\eta_2|e_\omega|(e^{|e_\omega|}-1)}{e^{|e_\omega|}(t_f-t)}\right] \quad (21)$$

Let $f_2(x) = \frac{x(e^x-1)}{e^x} = x(1-e^{-x}) \ge 0, \forall x$. Its derivative is

$$\dot{f}_2(x) = 1 - e^{-x} + xe^{-x}$$
$$\begin{cases} \dot{f}_2(x) > 0, \text{when } x > 0 \\ \dot{f}_2(x) < 0, \text{when } x < 0 \\ \dot{f}_2(x) = 0, \text{when } x = 0 \end{cases} \quad (22)$$

From (21) and the inequality $f_2(x) \ge 0, \forall x$, we obtain

$$\dot{V}_d \le -2\frac{\eta_1|e_v|(e^{|e_v|}-1)}{e^{|e_v|}(t_f-t)} \quad (23)$$

and

$$\dot{V}_d \le -2\frac{\eta_2|e_\omega|(e^{|e_\omega|}-1)}{e^{|e_\omega|}(t_f-t)} \quad (24)$$

Obviously, $V_d = e_v^2 + e_w^2 \le 2\max(|e_v|,|e_w|)^2$. This implies that $\sqrt{\frac{V_d}{2}} \le \max(|e_v|,|e_w|)$, so $\sqrt{\frac{V_d}{2}} \le |e_v|$ or $\sqrt{\frac{V_d}{2}} \le |e_w|$. Without loss of generality, it can be assumed that $|e_v| \ge |e_w|$, thus $\sqrt{\frac{V_d}{2}} \le |e_v|$. Combine (22) with (23) to get

$$\dot{V}_d \le -2\frac{\eta_1\sqrt{\frac{V_d}{2}}\left(e^{\sqrt{\frac{V_d}{2}}}-1\right)}{e^{\sqrt{\frac{V_d}{2}}}(t_f-t)} \quad (25)$$

Denote $\xi = \sqrt{\frac{V_d}{2}}$, so $\dot{\xi} = \frac{1}{2}\frac{\dot{V}_d}{2\sqrt{\frac{V_d}{2}}} = \frac{1}{4}\frac{\dot{V}_d}{\xi}$. Substitute this into Eq. (25) to have

$$4\xi\dot{\xi} \le -2\frac{\eta_1\xi(e^{\xi}-1)}{e^{\xi}(t_f-t)} \Rightarrow \dot{\xi} \le -\frac{\eta_1'(e^{\xi}-1)}{e^{\xi}(t_f-t)} \quad (26)$$

where $\eta_1' = \frac{\eta_1}{2}$.

According to [7], we have $\xi = 0, \forall t \ge t_f$, so $V_d = 0, \forall t \ge t_f$ or $\mathbf{e_v} = 0, \forall t \ge t_f$. Similarly, it can be proved for the case $|e_w| \ge |e_v|$. Note that $\eta_1' \ge 1$, it means $\eta_1 \ge 2$ and $\eta_2 \ge 2$.

(ii) As $t \ge t_f$

We have $\mathbf{u} = \dot{\mathbf{v}}_c$, so $\dot{\mathbf{v}} = \dot{\mathbf{v}}_c$, thus $\mathbf{v}(t) - \mathbf{v}(t_f) = \mathbf{v}_c(t) - \mathbf{v}_c(t_f)$, but $\mathbf{v}(t_f) = \mathbf{v}_c(t_f)$, this implies $\mathbf{v}(t) = \mathbf{v}_c(t)$.

2.4. Disturbance estimator

In this section, a disturbance estimator [8] will be applied for the system (7) with model uncertainty and input disturbance. This disturbance estimator is utilized for the following system:

$$\dot{\mathbf{x}} = \mathbf{\Phi}(\mathbf{x},t)\mathbf{x} + \mathbf{H}(\mathbf{x},t)(\mathbf{u}+\mathbf{d}) \quad (27)$$

A reference model with sampling time δ is given as:

$$\mathbf{z_k} = \mathbf{\Phi_k^z z_{k-1}} + \mathbf{H_k^x}(\mathbf{u} - \hat{\mathbf{d}}_{\mathbf{k-1}}). \quad (28)$$

Then, the input disturbance will be estimated as follows

$$\hat{\mathbf{d}}_\mathbf{k} = [(\mathbf{H}_\mathbf{k}^x)^\mathbf{T}\mathbf{H}_\mathbf{k}^x]^{-1}[(\mathbf{H}_\mathbf{k}^x)^\mathbf{T}(\mathbf{x}_\mathbf{k} - \mathbf{z}_\mathbf{k} - \mathbf{\Phi}_\mathbf{k}^x\mathbf{x}_{\mathbf{k-1}} + \mathbf{\Phi}_\mathbf{k}^z\mathbf{z}_{\mathbf{k-1}}), \tag{29}$$

where

$$\begin{gathered} \mathbf{\Phi}_\mathbf{k}^x = \mathbf{I} + \delta\mathbf{\Phi}(\mathbf{x}_{\mathbf{k-1}}, t_k) \\ \mathbf{H}_k^x = \delta\mathbf{H}(\mathbf{x}_{\mathbf{k-1}}, t_k) \\ \mathbf{\Phi}_k^z = \mathbf{I} + \delta\mathbf{\Phi}(\mathbf{z}_{\mathbf{k-1}}, t_k) \end{gathered} \tag{30}$$

To compensate the model uncertainty and input disturbance of the system (7), it is rewritten as

$$\left(\overline{\mathbf{M}}(\mathbf{q}) + \Delta\overline{\mathbf{M}}(\mathbf{q})\right)\dot{\mathbf{v}} + \left(\overline{\mathbf{V}}(\mathbf{q},\dot{\mathbf{q}}) + \Delta\overline{\mathbf{V}}(\mathbf{q},\dot{\mathbf{q}})\right)\mathbf{v} = \overline{\mathbf{B}}(\boldsymbol{\tau} + \Delta\boldsymbol{\tau}), \tag{31}$$

with $\Delta\overline{\mathbf{M}}(\mathbf{q})$, $\Delta\overline{\mathbf{V}}(\mathbf{q},\dot{\mathbf{q}})$ and $\Delta\boldsymbol{\tau}$ represent the model uncertainty and input disturbance, respectively. Denote $\mathbf{\Phi}(\mathbf{v},t) = -\overline{\mathbf{M}}^{-1}\overline{\mathbf{V}}$, $\mathbf{H}(\mathbf{v},t) = \overline{\mathbf{M}}^{-1}\overline{\mathbf{B}}$ and $\mathbf{d} = \Delta\boldsymbol{\tau} - \overline{\mathbf{B}}^{-1}\left(\Delta\overline{\mathbf{M}}\dot{\mathbf{v}} + \Delta\overline{\mathbf{V}}\mathbf{v} + \overline{\mathbf{F}}(\mathbf{v})\right)$, then, the system is rewritten as.

$$\dot{\mathbf{v}} = \mathbf{\Phi}(\mathbf{v},t)\mathbf{v} + \mathbf{H}(\mathbf{v},t)(\boldsymbol{\tau} + \mathbf{d}). \tag{32}$$

Finally, the disturbance estimator (29) will be applied for the system (32). In combination with the controllers (12) and (14), the real input to the system (31) is

$$\boldsymbol{\tau} - \hat{\mathbf{d}}_\mathbf{k} = f(\mathbf{q},\dot{\mathbf{q}},\mathbf{v},\mathbf{u}) = \overline{\mathbf{B}}^{-1}(\mathbf{q})\left[\overline{\mathbf{M}}(\mathbf{q})(\mathbf{C} + \dot{\mathbf{v}}_\mathbf{c}) + \overline{\mathbf{V}}(\mathbf{q},\dot{\mathbf{q}})\mathbf{v}\right] - \hat{\mathbf{d}}_\mathbf{k}, \tag{33}$$

for all time such that $k\delta \le t < (k+1)\delta$ with $k = 0, 1, 2, ...$

In the next section, the arbitrary convergence time controller (14) in combination with disturbance estimator (29), which is also (33), will be verified through numerical simulations.

3. Numerical simulations

Nominal values of the WMR's parameters are given as follows: $m = 4\ (kg)$, $I = 2{,}5\ (kg.m^2)$, $R = 0{,}15\ (m)$, $r = 0{,}03 (m)$, and $d = 0{,}15 (m)$. These values are used for computing control signal and estimating disturbance. To create the model's uncertainty for the WMR, the parameters for simulating the plant are increased as follows: $m = 6 (kg)$, $I = 5 (kg.m^2)$, $R = 0{,}18\ (m)$, $r = 0{,}036\ (m)$ and $d = 0{,}18\ (m)$.

A square reference trajectory will be used with following desired longitudinal and angular velocities.

$$v_r = \begin{cases} 0.785\ (\mathrm{m/s}), & (0 + 10n) < t \le (3{,}146 + 10n) \\ 1\ (\mathrm{m/s}), & t > (3.146 + 10\mathrm{n}) \end{cases}$$

$$\omega_r = \begin{cases} 0.5\ (\mathrm{rad/s}), & (0 + 10n) < t \le (3{,}146 + 10n) \\ 0\ (\mathrm{rad/s}), & t > (3.146 + 10\mathrm{n}) \end{cases}$$

with $n = 0, 1, 2, 3$. The initial position of the reference trajectory is set at $\mathbf{q}_\mathbf{r}(0) = [x_r(0), y_r(0), \theta_r(0)]^T = [0, 0, 0]^T$ and that of the WMR is given as $\mathbf{q}(0) = [x(0), y(0), \theta(0)]^T = [2, 2, pi]^T$.

The unknown input disturbance is given for simulation as $\Delta\boldsymbol{\tau} = \begin{bmatrix} 2\sin(10t) + 2\cos(5t) \\ 1\sin(5t) + 3\cos(10t) \end{bmatrix}$ and $t_f = 5\ (s)$.

Fig. 3 shows numerical simulation results. The proposed method provided very small tracking control errors in comparison with the case that the new method without disturbance estimator was used.

Some different values of the desired time t_f were also used for simulation. It can draw a conclusion that as t_f is decreased the control signal $\boldsymbol{\tau}$ tends to be bigger for the starting time while the tracking errors are similar.

4. Conclusion and future work

This work proposed a tracking controller for WMRs with model uncertainty and input disturbance, in which an arbitrary convergence time controller for the velocity control loop was associated with a disturbance estimator to eliminlate the effect of model uncertainty and input disturbance and force the velocity errors to converge to zero in arbitrary finite time. The numerical simulation proved the effective of the proposed method.

Future works focus on stability analysis of the overall system and implement on real WMRs to assess the proposed method on practical WMRs.

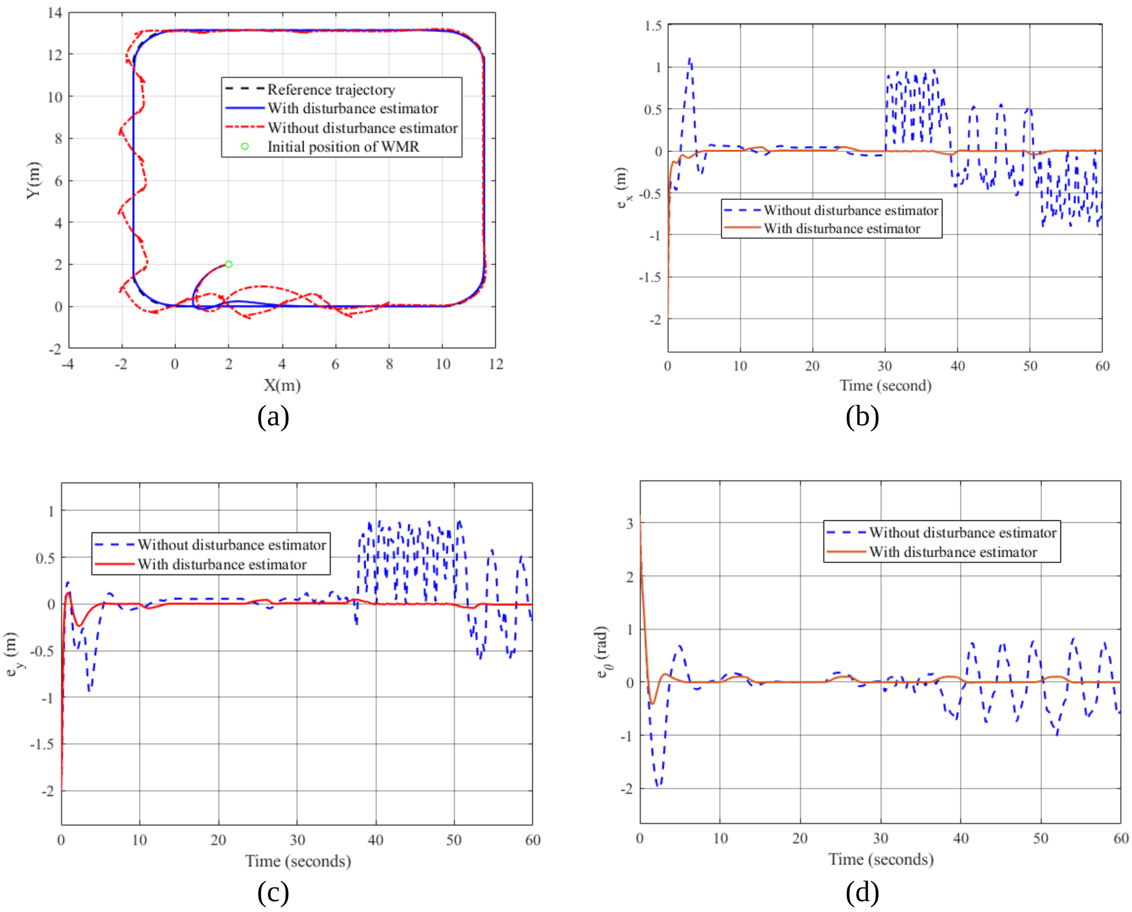

Figure 3. *Comparison of the proposed method to the case without disturbance estimator. (a) WMR's trajectories, (b) control errors for x, (c) control errors for y, (d) control errors for orientation angle θ.*

REFERENCES

[1] E. A. Oyekanlu et al., "A Review of Recent Advances in Automated Guided Vehicle Technologies: Integration Challenges and Research Areas for 5G-Based Smart Manufacturing Applications," in *IEEE Access*, vol. 8, pp. 202312-202353, 2020, doi: 10.1109/ACCESS.2020.3035729.

[2] R. Fierro and F. L. Lewis, "Control of a nonholonomic mobile robot: backstepping kinematics into dynamics," in *Proceedings of 1995 34th IEEE Conference on Decision and Control*, Dec. 1995, vol. 4, pp. 3805–3810 vol.4. doi: 10.1109/CDC.1995.479190.

[3] C.-Y. Chen, T.-H. S. Li, Y.-C. Yeh, and C.-C. Chang, "Design and implementation of an adaptive sliding-mode dynamic controller for wheeled mobile robots," *Mechatronics*, vol.19, no. 2, 2009, pp. 156-166.

[4] H. Yang, X. Fan, P. Shi and C. Hua, "Nonlinear Control for Tracking and Obstacle Avoidance of a Wheeled Mobile Robot with Nonholonomic Constraint," in *IEEE*

Transactions on Control Systems Technology, vol. 24, no. 2, pp. 741-746, March 2016, doi: 10.1109/TCST.2015.2457877.

[5] L. Xin, Q. Wang, J. She, and Y. Li, "Robust adaptive tracking control of wheeled mobile robot," *Robotics and Autonomous Systems*, vol. 78, pp. 36-48, 2016.

[6] H. Xiao et al., "Robust Stabilization of a Wheeled Mobile Robot Using Model Predictive Control Based on Neurodynamics Optimization," in *IEEE Transactions on Industrial Electronics*, vol. 64, no. 1, pp. 505-516, Jan. 2017, doi: 10.1109/TIE.2016.2606358.

[7] A. K. Pal, S. Kamal, S. K. Nagar, B. Bandyopadhyay, and L. Fridman, "Design of controllers with arbitrary convergence time," *Automatica*, vol. 112, p. 108710, Feb. 2020, doi: 10.1016/j.automatica.2019.108710.

[8] P. D. Nguyen and N. H. Nguyen, "Adaptive control for nonlinear non-autonomous systems with unknown input disturbance," *Int. J. Control*, pp. 1–11, Aug. 2021, doi: 10.1080/00207179.2021.1974571.