PERFORMANCE EVALUATION OF THE REACHING LAWS IN SLIDING MODE CONTROL ON THE MASS SPRING DAMPER SYSTEM

ĐÁNH GIÁ HIỆU QUẢ CỦA CÁC LUẬT TIẾP CẬN TRONG ĐIỀU KHIỂN TRƯỢT HỆ THỐNG GIẢM XÓC - VẬT - LÒ XO

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ABSTRACT

In this paper, the reaching laws in the sliding mode control (SMC) will be evaluated. The reaching laws include the constant rate reaching law, the exponential reaching law, and the power rate reaching law. A mass spring damper system (MSD) is chosen to test and evaluate the performance of the proposed controller. The MSD system has been applied in most dynamic suspension systems to provide more reliability of the design requirements such as increasing the factors of safety or absorbing the impact forces. The SMC based on the reaching laws will be designed to guarantee the real position of the MSD system following the reference. Simulation results in MATLAB/Simulink show that the SMC provides better performance than the traditional PID controller, and the SMC based on the exponential reaching law is suitable to control the MSD system with the overshoot is about 0.154(%), the steady-state error is eliminated, the settling time is 0.4043(s), and the rising time achieves 0.2248(s).

Keywords: *Mass spring damper system, sliding mode control, position control, reaching laws, MATLAB/Simulink.*

TÓM TẮT

Trong bài báo này, các luật tiếp cận trong điều khiển trượt sẽ được đánh giá. Các luật tiếp cận bao gồm luật tiếp cận tốc độ hằng, luật tiếp cận tốc độ hàm mũ và luật tiếp cận tốc độ hàm lũy thừa. Hệ thống giảm xóc - vật - lò xo được chọn để kiểm tra và đánh giá hiệu quả của bộ điều khiển đề xuất. Hệ thống này đã được áp dụng trong hầu hết các hệ thống treo động lực học để cung cấp độ tin cậy cao hơn cho các yêu cầu thiết kế như tăng hệ số an toàn hoặc hấp thụ lực tác động. Bộ điều khiển trượt dựa vào các luật tiếp cận sẽ được thiết kế để đảm bảo vị trí thực tế của hệ thống bám theo vị trí tham khảo. Các kết quả mô phỏng trong MATLAB/Simulink cho thấy rằng bộ điều khiển trượt đạt hiệu quả tốt hơn bộ điều khiển PID truyền thống; đồng thời, bộ điều khiển trượt dựa vào luật tiếp cận tốc độ hàm mũ phù hợp để điều khiển hệ thống với độ vọt lố khoảng 0.154(%), sai số xác lập được triệt tiêu, thời gian xác lập là 0.4043(s) và thời gian tăng là 0.2248(s).

Từ khóa: *Hệ thống giảm xóc - vật - lò xo, điều khiển trượt, điều khiển vị trí, luật tiếp cận, MATLAB/Simulink.*

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1. INTRODUCTION

Sliding mode control (SMC) technique is an interesting research approach for non-linear systems. This technique was firstly proposed and elaborated in early 1950s in the Soviet Union by Emelyanov and several co-researchers such as Utkins and Itkis. During the last decades' significant interest on the Variable Structure Control and the SMC have been developed in the control system research community [1]. The main advantage of the SMC is that, in the sliding mode, the behavior of the system can be only determined by the sliding surface dynamics, and the rest invariant components of the system can be considered in a class of parametric uncertainties and disturbances [2]. This invariant property makes the SMC achieving a robust control strategy that can be used for several practical control applications.

In recent years, the MSD system are widely used in different areas of engineering field applications [3]. The concept of the MSD system has been implemented on a vast number of practical fields, such as in the reduction of vibrations, in control system analysis, and in power generation [4]; in manipulator control, it serves as a shock absorber for the vehicle suspension system, or piezoelectric vibration energy harvester [5]. A number of publications on the MSD system is researched and developed for instance. The reference [3] proposed a control model for a MSD electromechanical system which compared a backstepping technique with a conventional proportional–derivative–integral (PID) controller to evaluate the performance of the control systems. In reference [4], the authors studied the entropy generation of a SMD mechanical system, under the conformable fractional operator definition. An issue of performance evaluation of two control schemes including proportional-derivate (PD) and Linear Quadratic Regulator (LQR) controllers for a coupled MSD is presented in [5]. In the reference [6, 7], a PID controller was used to control the MSD system. The metaheuristic algorithms namely multi-objective genetic algorithm (MOGA) and adaptive particle swarm optimization (APSO) algorithms are used in [8] to determine optimal gain parameters of linear and nonlinear PID controllers. The P, PI, PD, and PID controllers are used in [9] to design and comparatively analyze for MSD system. Those publications have been involved in comparison some control techniques with the SMC. However, there is a limit of studies on evaluating the effects of reaching laws on the SMC.

On the other hand, when chattering phenomenon occurs at signal control, the power driver circuits are susceptible to overheating, resulting in damage. Therefore, the performance evaluation of the SMC laws will allow to select an essential safe control method. In this paper, firstly, the SMC based on reaching laws will be designed to control the position of the MSD system so that the signal of the closed-loop system can track to the reference with the trajectory tracking errors converge to zero in finite time. Secondly, the efficiency of the SMC controller with different reaching laws will be assessed by its quality indicators in order to select a suitable SMC law to control the MSD system.

The rest of this paper is organized as follows: Section 2 presents the mathematical model of the MSD system; applying the SMC on the MSD system based on different reaching laws is given in Section 3; Section 4 shows the simulation results in MATLAB/Simulink and the conclusion is given in Section 5.

2. MATHEMATICAL MODEL OF THE MASS SPRING DAMPER SYSTEM

The MSD system consists of a mass, a spring and a damper. The spring with constant k serves to restore the mass to a neutral position. The damping element opposes the motion of the vibratory response with a proportional force to the velocity of the system that the proportional constant being a damping constant d. Let us consider the schematic diagram of the MSD presented in Figure 1. And Figure 2 gives the free body diagram of the system [10].

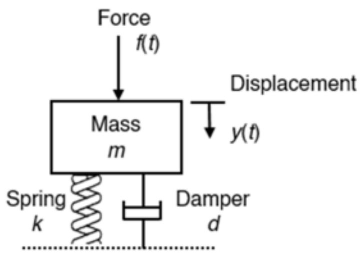

Figure 1. Schematic diagram of the MSD system [10]

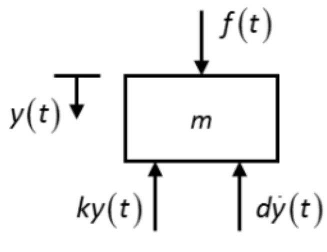

Figure 2. Free body diagram of the system [10]

Applying Newton's second law, we can state that:

$$\sum F_y(t) = ma \tag{1}$$

$$F(t) + F_D(t) + F_S(t) = ma \tag{2}$$

where:

y(t) is the position of object with the mass m.

$F_D(f)$ = -ky(t) is the spring force, and k is the spring constant (N/m).

$F_S(t)$ = -dy(t) is the viscous damping force and d is damper constant (Ns/m).

F(t) = f(t) is the denotes the external force.

$ma = m\ddot{y}(t)$ is the inertia force and m being the mass of object (kg).

Equation (2) can be rewritten as equation (3):

$$f(t) - ky(t) - d\dot{y}(t) = m\ddot{y}(t) \tag{3}$$

$$\Leftrightarrow m\ddot{y}(t) + d\dot{y}(t) + k(y) = F(t) \tag{4}$$

Equation (4) represents a mathematical model of the MSD system.

Let define state variables as follows:

$$x_1(t) = y(t) \tag{5}$$

$$x_2(t) = \dot{x}_1(t) = \dot{y}(t) \tag{6}$$

$$\Rightarrow \dot{x}_2(t) = \ddot{x}_1(t) = \ddot{y}(t) \tag{7}$$

Referring equations (5), (6) and (7) to equation (4), we have equation (8):

$$\dot{x}_2(t) = \ddot{y}(t) = \frac{1}{m}f(t) - \frac{d}{m}\dot{y}(t) - \frac{k}{m}y(t) \tag{8}$$

From equations (6) and (8), we have equations (9) and (10):

$$\dot{x}_1(t) = x_2(t) \tag{9}$$

$$\dot{x}_2(t) = -\frac{k}{m}x_1(t) - \frac{d}{m}x_2(t) + \frac{1}{m}f(t) \tag{10}$$

And the output equation becomes

$$y(t) = x_1(t) \tag{11}$$

or

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{d}{m} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} f(t) \tag{12}$$

and

$$y(t) = x_1(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \tag{13}$$

Equations (12) and (13) give a state - space representation of the MSD system.

3. SLIDING MODE CONTROL MSD SYSTEM BASED ON REACHING LAWS

3.1. SMC based on reaching laws

In general, a SMC based on reaching laws includes a reaching phase and a sliding phase. The reaching phase drives the system to stable manifold, the sliding phase drives the system slide to equilibrium. The typical reaching laws are given as follows [1]:

a. Reaching law with constant rate

$$\dot{s} = -\varepsilon \text{sign}(s), \varepsilon > 0 \tag{14}$$

where ε represents a constant rate.

This law forces the switching variable to reach the switching manifold s at a constant rate ε. The merit of this reaching law is its simplicity.

b. Exponential reaching law

$$\dot{s} = -\varepsilon \text{sign}(s) - ks, \ \varepsilon > 0, k > 0 \tag{15}$$

where $\dot{s} = -ks$ is an exponential term, and its solution is $s = s(0)e^{-kt}$.

In the exponential reaching law, for being to guarantee a faster convergence speed, especially when *s* is nearly to zero, the term $\dot{s} = -\varepsilon \text{sign}(s)$ is used.

c. Reaching law with power rate

$$\dot{s} = -k|s|^{\alpha} \text{sign}(s), k > 0, 0 < \alpha < 1 \tag{16}$$

This reaching law increases the reaching speed when the state is far away from the switching manifold, but reduces the rate when the state is near the manifold. The result is a fast reaching and low chattering reaching mode.

3.2. SMC controller design based on reaching laws for the MSD system

The SMC controller consists of equivalent control and switching control.

$$u_{SMC}(t) = u_{eq}(t) + u_{sw}(t) \tag{17}$$

Now, the equivalent control and switching control can be designed:

- Switching function

The sliding surface is chosen as equation (18):

$$S(t) = \left(\frac{d}{dt} + \lambda\right)^{n-1} e(t) \tag{18}$$

Since the order of the system is 2, then:

$$S(t) = \dot{e}(t) + \lambda e(t) \tag{19}$$

Where λ > 0 is performance parameter which guaranteed the stability of the system.

The tracking error and its derivative value is

$$e(t) = y_d(t) - y(t)$$

$$\dot{e}(t) = \dot{y}_d(t) - \dot{y}(t)$$

Where $y_d(t)$ is the reference position, y(t) is the reality position.

The first derivative of equation (19), we have equation (20):

$$\dot{S}(t) = \ddot{e}(t) + \lambda\dot{e}(t) = \lambda\dot{e}(t) + \ddot{y}_d(t) - \ddot{y}(t) \tag{20}$$

Substituting equation (10) in equation (20), we have:

$$\dot{S}(t) = \lambda\dot{e}(t) + \ddot{y}_d(t) + \frac{k}{m}x_1(t) + \frac{d}{m}x_2(t) - \frac{1}{m}f(t) \tag{21}$$

On sliding surface: $S(t) = 0 \rightarrow \dot{S}(t) = 0$

Equivalent control can be found:

$$\dot{S}(t) = \lambda\dot{e}(t) + \ddot{y}_d(t) + \frac{k}{m}x_1(t) + \frac{d}{m}x_2(t) - \frac{1}{m}f(t) = 0$$

$$u_{eq}(t) = m\lambda\dot{e}(t) + m\ddot{y}_d(t) + kx_1(t) + dx_2(t) \tag{22}$$

- The reaching law with constant rate (i.e. switching control) is described as equation (23):

$$u_{sw}(t) = W\text{sign}(S(t)) \tag{23}$$

Where W > 0 is selected sufficiently large. A larger value of W allows a faster the trajectory converges to the sliding surface.

The signum function has its value equally distributed in both positive and negative planes from its origin. The mathematical model of the signum function is described as equation (24):

$$\text{sign}(S(t)) = \begin{cases} 1, & S(t) > 0 \\ 0, & S(t) = 0 \\ -1, & S(t) < 0 \end{cases} \tag{24}$$

The graph of the signum function is shown as Figure 3.

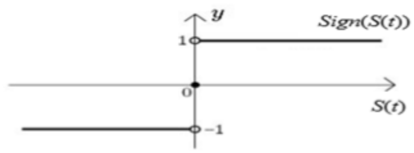

Figure 3. The image of the signum function

Combine equations (22) and (23), we have the SMC controller as follow equation (25):

$$\begin{aligned} u_{SMC}(t) &= m\lambda\dot{e}(t) + m\ddot{y}_d(t) + kx_1(t) \\ &\quad + dx_2(t) + W\text{sign}(S(t)) \end{aligned} \tag{25}$$

To prove the stability, the Lyapunov function can be defined by equation (26):

$$V(t) = \frac{1}{2}S^2(t) \tag{26}$$

$$\dot{V}(t) = S(t)\dot{S}(t)$$

$$= S(t)\begin{pmatrix} \lambda\dot{e}(t) + \ddot{y}_d(t) + \frac{k}{m}x_1(t) + \frac{d}{m}x_2(t) \\ -\frac{1}{m}\left(m\lambda\dot{e}(t) + m\ddot{y}_d(t) + kx_1(t) + dx_2(t) + W\text{sign}(S(t))\right) \end{pmatrix}$$

$$= S(t)\left(-W\text{sign}(S(t))\right) = -WS(t)\text{sign}(S(t))$$

$$= -W|S(t)| < 0$$

We can see that the sliding mode function S(t) will tend to zero exponentially with W value.

Similar to above, the exponential reaching law and the power rate reaching law are described as equations (27) and (28):

$$u_{SMC}(t) = m\lambda\dot{e}(t) + m\ddot{y}_d(t) + kx_1(t) + dx_2(t) + W\text{sign}(S(t)) + kS(t) \quad (27)$$

Where $W > 0, k > 0$.

$$u_{SMC}(t) = m\lambda\dot{e}(t) + m\ddot{y}_d(t) + kx_1(t) + dx_2(t) + k|S(t)|^{\alpha}\,\text{sign}(S(t)) \quad (28)$$

where $k > 0, 0 < \alpha < 1$.

4. SIMULATION RESULTS IN MATLAB/ SIMULINK

The structure of the SMC for the MSD is presented in Figure 4.

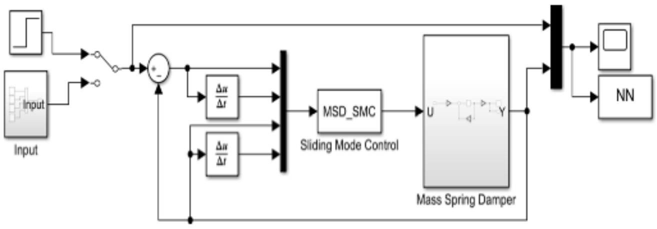

Figure 4. The structure of the SMC for the MSD

The parameters of the MSD and SMC are shown as Table 1.

Table 1. The parameters of the MSD and SMC

Parameters	Mass of the system	Spring constant	Damper Coefficient	W	k	α	λ
Values	1	1	2	32	200	0.01	10
Unit	kg	N/m	Ns/m				

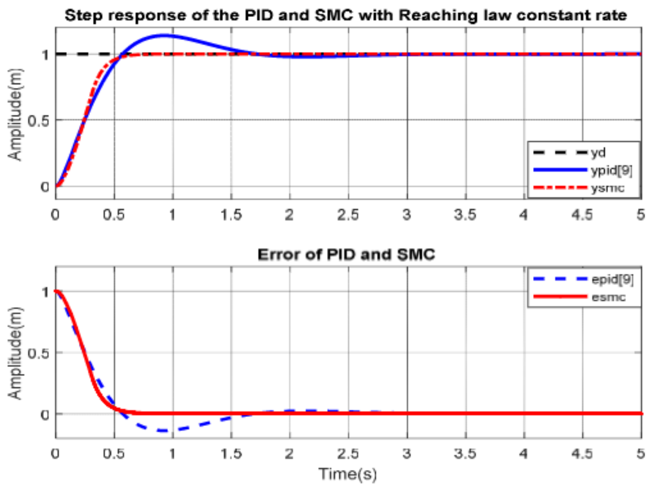

Figure 5. The response and error of the SMC based on constant rate reaching law

The step response of the SMC for the MSD based on reaching laws is compared with a traditional PID controller with the values of $k_P = 10$, $k_D = 2$, $k_I = 5$, respectively [9]. Figures 5, 6 and 7 present the response and the error of the SMC based on 3 kinds of reaching laws mentioned above. In Figure 5, the step response of the SMC converges to the reference with the settling time is 0.5819 (s), the overshoot is 1.04 (%) and the rise time is 0.3298 (s); Figure 6 is 0.4043 (s), 0.154 (%) and 0.2248 (s); And Figure 7 is 0.5033 (s), 2.33 (%) and 0.2745, respectively. These values are indicated in Table 2. In Table 2, we see that the performance of the SMC is better than the classical PID.

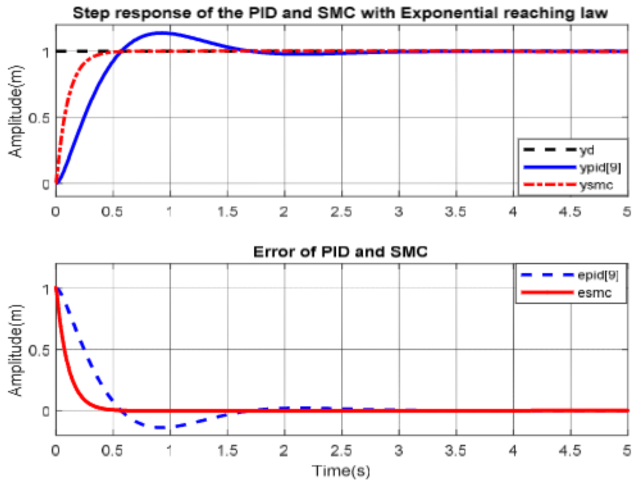

Figure 6. The response and error of the SMC based on exponential reaching law

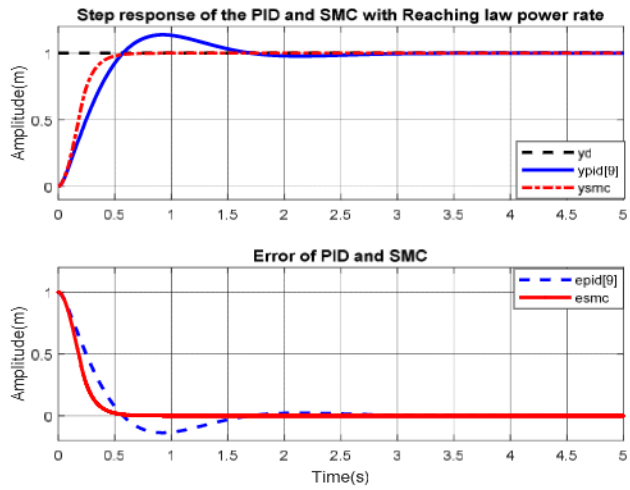

Figure 7. The response and error of the SMC based on power rate reaching law

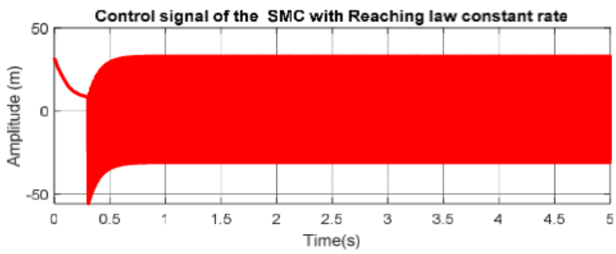

Figure 8. Control signal of the SMC with constant rate reaching law

Table 2. System responses with the SMC based on different reaching laws

Controller		Percent of overshoot (%)	Steady-state error (m)	Rise time (s)	Settling time (s)
SMC	Reaching law with constant rate	1.04	0	0.3298	0.5819
	Exponential reaching law	0.154	0	0.2248	0.4043
	Reaching law with power rate	2.33	0	0.2745	0.5033
PID [9]		13.5	0	0.442	2.46

The control signals of the SMC based on reaching law with constant rate, exponential reaching law and reaching law power rate are presented in Figures 8, 9 and 10, respectively. An oscillation frequency of the control signal in Figures 8 and 9 is greater than in Figure 10. It shows that, the SMC based on the constant rate reaching law and the power rate reaching law cause more chattering than the exponential reaching law.

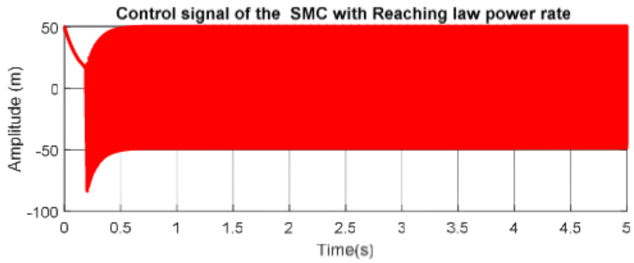

Figure 9. Control signal of the SMC with power rate reaching law

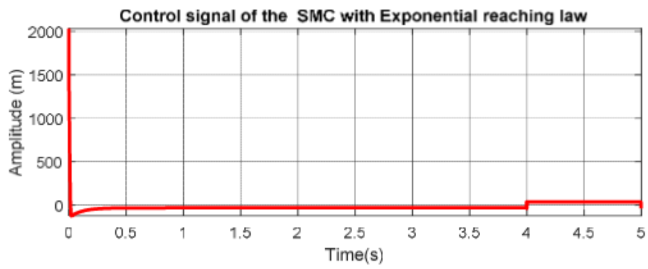

Figure 10. Control signal of the SMC with exponential reaching law

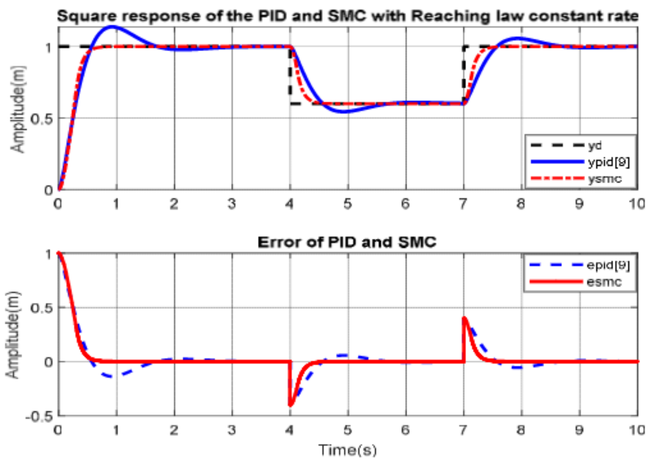

Figure 11. The square response & error of the SMC based on constant rate reaching law

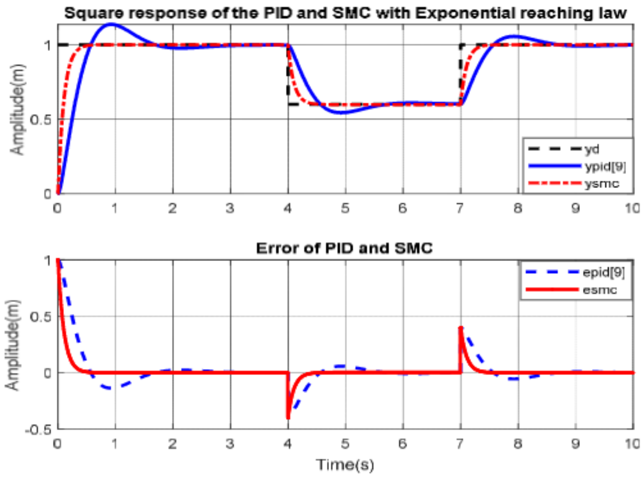

Figure 12. The square response & error of the SMC based on exponential reaching law

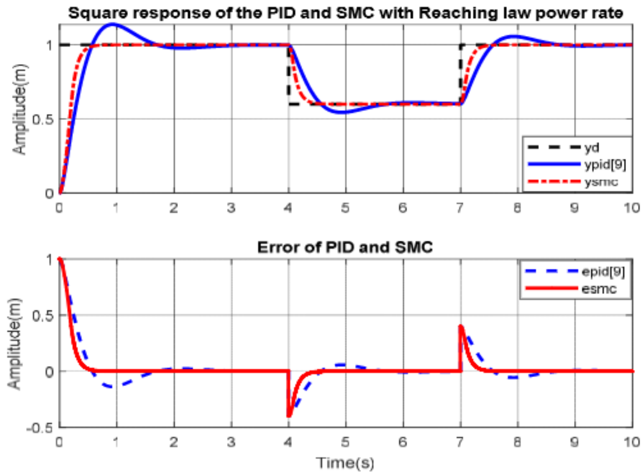

Figure 13. The square response & error of the SMC based on power rate reaching law

The square response of the SMC for the MSD based on three different reaching laws are presented in Figures 11, 12 and 13. These responses are also compared with the traditional PID controller and we can see that the performance of the SMC is also better than the classical PID. Looking the square responses in Figures 11, 12 and 13, we see that the error of the SMC based on reaching laws converging to zero faster than the PID controller.

5. CONCLUSION

In this research, three kinds of reaching laws in SMC are designed and applied in position control of the MSD system. The traditional PID controller is also applied to compare and evaluate the performance of the quality control. The simulation results in MATLAB/Simulink indicate that the performance of the proposed controller is better than the traditional PID. The reality position of the system converges to the reference in the finite time. The quality indicators of the SMC with exponential reaching law is better than the reaching law with constant rate and the reaching law with power rate; the oscillation frequency of control signal is smaller. In further work, the proposed controller should be tested on the real MSD system instead of the simulation only.

REFERENCES

[1]. Jinkun Liu, 2017. *Sliding mode control using MATLAB*. Published by Elsevier Inc.

[2]. Devika K. B., Susy Thomas, 2017. *Power Rate Exponential Reaching Law for Enhanced Performance of Sliding Mode Control*. International Journal of Control, Automation and Systems, 15/6, 2636 - 2645.

[3]. Munaf Fathi Badr, Ekhlas Hameed Karam, Noor Mohammaed Mjeed, 2020. *Control design of damper mass spring system based on backstepping controller scheme*. International Review of Applied Sciences and Engineering, 11/2, 181 - 187.

[4]. Jorge M. Cruz-Duarte, J. Juan Rosales-García, C. Rodrigo Correa-Cely, 2020. *Entropy Generation in a Mass-Spring-Damper System Using a Conformable Model*. Symmetry, 12/3,1 - 11.

[5]. Okubanjo, A.A, Oyetola, O.K, Ade-Ikuesan, O.O, Olaluwoye, O.O, Alao, P.O, 2018. *Performance Evaluation of PD and LQR Controller for Coupled Mass Spring Damper System*. Futo Journal Series (FUTOJNLS), 4/1, 199 - 210.

[6]. P. S. Contreras Ortiz, B. R. Naula Dutan, L. G. Torres Jaramillo, R. S. Ávila-Campoverde, M. A. Carpio Aleman, J. C. Montesdeoca-Contreras, J. A. Morales-Garcia, 2014. *Linearization of Mass Spring Damper System for Applying Linear Control PID Techniques*. IEEE International Autumn Meeting on Power, Electronics and Computing (ROPEC), 1 - 5.

[7]. Shabnam Armaghan, Arefeh Moridi, Ali Khaki Sedigh, 2011. *Design of a Switching PID Controller for a Magnetically Actuated Mass Spring Damper*. Proceedings of the World Congress on Engineering, 3, 2190 - 2193.

[8]. Sudarshan K. Valluru, Madhusudan Singh, 2017. *Metaheuristic Tuning of Linear and Nonlinear PID Controllers to Nonlinear Mass Spring Damper System*. International Journal of Applied Engineering Research, 12/10, 2320 - 2328.

[9]. Kankariya Ravindra, Kulkarni Yogesh, Gujrathi Ankit, 2018. *Comparative Analysis of P, PI, PD, PID Controller for Mass Spring Damper System using Matlab Simulink*. International Journal for Research in Engineering Application & Management (IJREAM), Special Issue, 668 - 672.

[10]. Ejiroghene Kelly Orhorhoro, Monday Erhire Onogbotsere, Aniekan Essienubong Ikpe, 2016. *Simulation of a mass spring damper model in phase variable*. ELK Asia Pacific Journal of Mechanical Engineering Research, 2/2, 1 – 16.

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