NEW RESULTS ON FINITE-TIME GUARANTEED COST CONTROL OF LINEAR UNCERTAIN CONFORMABLE FRACTIONAL-ORDER SYSTEMS

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ABSTRACT

In this paper, we investigate the problem of finite-time guaranteed cost control of linear uncertain conformable fractional order systems. Firstly, a new cost function is defined. Then, by using some properties of conformable fractional calculus, some new sufficient conditions for the design of a state feedback controller that makes the closed-loop systems finite-time stable and guarantees an adequate cost level of performance is derived via linear matrix inequalities, therefore can be efficiently solved by using existing convex algorithms. A numerical example is given to illustrate the effectiveness of the proposed method.

Keyword: *problem; finite-time guaranteed cost control; linear uncertain conformable fractional order; systems; cost function.*

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MỘT VÀI KẾT QUẢ MỚI VỀ BÀI TOÁN ĐẢM BẢO CHI PHÍ ĐIỀU KHIỂN TRONG THỜI GIAN HỮU HẠN CỦA HỆ PHƯƠNG TRÌNH VI PHÂN TUYẾN TÍNH PHÂN THỨ PHÙ HỢP

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TÓM TẮT

Trong bài báo này, chúng tôi nghiên cứu bài toán đảm bảo chi phí điều khiển trong thời gian hữu hạn của hệ phương trình vi phân tuyến tính phân thứ phù hợp. Trước hết, chúng tôi đưa ra một định nghĩa về hàm chi phí. Sau đó, bằng cách sử dụng một số tính chất về giải tích phân thứ, một điều kiện đủ cho việc thiết kế một điều khiển ngược tuyến tính đảm bảo cho hệ đóng tương ứng không những ổn định hữu hạn thời gian mà còn đảm bảo hàm chi phí hữu hạn trong khoảng thời gian đó. Các điều kiện nhận được đều dưới dạng các bất đẳng thức ma trận tuyến tính và có thể giải số được một cách hiệu quả bằng các thuật toán lồi đã có. Một ví dụ số được đưa ra để minh họa cho sự hiệu quả cho kết quả của chúng tôi.

Từ khóa: *bài toán; đảm bảo chi phí điều khiển trong hữu hạn thời gian; hệ phương trình; vi phân tuyến tính phân thứ phù hợp; hàm chi phí.*

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1. Introduction

Recently, a new definition of local fractional (non-integer order) derivative which is called the conformable fractional derivative was introduced in [1]. Some well-behaved properties of the conformable fractional calculus such as chain rules, exponential functions, Gronwall's inequality, fractional integration by parts were derived in [2]. The interest in the conformable derivative has been increasing in the recent years because it has numerous applications in science and engineering. By using Lyapunov function, the problems of stability and asymptotic stability of conformable fractional-order nonlinear systems were studied in [3]. Necessary and sufficient conditions for the asymptotic stability of the positive linear conformable fractional-order systems were reported in [4]. On the other hand, from the view of engineering, it is desirable to design a controller such that the closed-loop system is finite-time stable and an adequate level of system performance is guaranteed. Some interesting results on the problem of finite-time guaranteed cost control for integer-order systems were derived in [5, 6, 7, 8]. Although there have been some works dedicated to Lyapunov stability and finite-time stability of conformable fractional-order systems, there are no results on finite-time control of uncertain conformable fractional-order systems. The main aim of this paper is to fill this gap.

In this paper, we present a novel approach to solve the problem of finite-time guaranteed cost control for linear uncertain conformable fractional-order systems. Consequently, some new explicit criteria for the problem are derived via linear matrix inequalities, which therefore can be efficiently solved by using existing convex algorithms. A numerical example is given to demonstrate of the feasibility and the effectiveness of our obtained results.

Notations: The following notations will be used in this paper: $\mathbb{R}^n$ denotes the n -dimensional linear real vector space with the Euclidean norm $\|.\|$ given by $\|x\| = \sqrt{x_1^2 + x_2^2 + ... + x_n^2}$, $x = (x_1, x_2, ..., x_n)^T \in \mathbb{R}^n$. For a real matrix A, $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ denote the maximal and the minimal eigenvalue of A, respectively.

2. Preliminaries and Problem statement

Firstly, we recall some definitions and traditional results, which are essential in order to derive our main results in this paper.

Definition 2.1. ([1]) For any $\alpha \in (0; 1]$, the conformable fractional derivative $T_{t_0}^{\alpha}(f(t))$ of the function $f(t)$ of order α is defined by

$$T_{t_0}^{\alpha}(f(t)) = \lim_{\varepsilon \to 0} \frac{f(t + \varepsilon(t - t_0)^{1-\alpha}) - f(t)}{\varepsilon}$$

If $t_0 = 0$, then $T_{t_0}^{\alpha}(f(t))$ has the form

$$T_0^{\alpha}(f(t)) = \lim_{\varepsilon \to 0} \frac{f(t + \varepsilon t^{1-\alpha}) - f(t)}{\varepsilon}$$

If the conformable fractional derivative $f(t)$ of order α exists on $(t_0, +\infty)$, then the function $f(t)$ is said to be α-differentiable on the interval $(t_0, +\infty)$.

Deffinition 2.2. ([1]) Let $\alpha \in (0, 1]$. The conformable fractional integral starting from a point t_0 of a function $f : [t_0, +\infty) \to \mathbb{R}$ of order α is defined as

$$I_{t_0}^{\alpha}(f(t)) = \int_{t_0}^{t} (s - t_0)^{\alpha - 1} f(s) ds, \quad t > 0$$

If $t_0 = 0$, then $I_{t_0}^{\alpha}(f(t))$ has the form

$$I_0^{\alpha}(f(t)) = \int_0^t s^{\alpha-1} f(s) ds, \quad t > 0$$

In the case $t_0 = 0$, we will denote $I_0^{\alpha}(f(t)) = I^{\alpha}(f(t))$

Lemma 2.3. ([2]) Let the function $f : [t_0, +\infty) \to \mathbb{R}$ be differentiable and $\alpha \in (0,$

1]. Then for all $t > t_0$ we have $I_{t_0}^{\alpha}(T_{t_0}^{\alpha} f(t)) = f(t) - f(t_0).$

Lemma 2.4. ([3]) Let $x:[t_0,+\infty)\to\mathbb{R}$ such that $T_{t_0}^{\alpha}x(t)$ exists on $[t_0,+\infty)$ and P is a symmetric positive definite matrix. Then $T_{t_0}^{\alpha}x^T(t)Px(t)$ exists on $[t_0,+\infty)$ and $T_{t_0}^{\alpha}x^T(t)Px(t) = 2x^T(t)PT_{t_0}^{\alpha}x^T(t), \forall t > t_0$.

Let us now consider the following uncertain conformable fractional-order system:

$$\begin{cases} T^{\alpha}x(t) = [A+\Delta A(t)]x(t) + [D+\Delta D(t)]\omega(\mathrm{t}) \\ \qquad\qquad +[B+\Delta B(t)]u(t) \\ x(0) = x_0 \end{cases} \quad (1)$$

where $\alpha \in (0, 1]$, $x(t)\in\mathbb{R}^n$ is the state, $\mathrm{u}(t)\in\mathbb{R}^m$ is the control, $\omega(t)\in\mathbb{R}^p$ is the disturbance, $x_0\in\mathbb{R}^n$ is the initial condition, A, D, B are known real constant matrices of appropriate dimensions. We make the following assumptions throughout this paper:

(H1) $\Delta A(t) = E_aF_a(t)H_a, \Delta D(t) = E_dF_d(t)H_d,$
$\Delta B(t) = E_bF_b(t)H_b$

where $E_a, E_d, E_b, H_a, H_d, H_b$ are known real constant matrices of appropriate dimensions, $F_a(t), F_d(t), F_b(t)$ are unknown real time-varying matrices satisfying

$F_a^T(t)F_a(t)\le I,\ F_d^T(t)F_d(t)\le I,$
$F_b^T(t)F_b(t)\le I, \forall \mathrm{t}\ge 0$.

(H2) The disturbance $\omega(t)\in\mathbb{R}^p$ satisfies the following condition

$$\exists d > 0: \omega^T(t)\omega(\mathrm{t}) \le d, \forall t\in[0,T_f]. \quad (2)$$

Given a positive number $T_f > 0$. Associated with the system (1) is the following quadratic cost function:

$$J(u) = \int_0^{T_f} s^{\alpha-1}\left(x^T(s)Q_1x(s) + u^T(s)Q_2u(s)\right)ds, \quad (3)$$

where $Q_1\in\mathbb{R}^{nxm}, Q_2\in\mathbb{R}^{nxm}$ are given symmetric positive definite matrices.

Remark 1. It should be noted that when $\alpha = 1$ the quadratic cost function (3) is turned into the definition of cost function in integer-order systems which was considered in the literature [6].

The unforced system of the system (1) can be expressed as

$$\begin{cases} T^{\alpha}x(t) = [A+\Delta A(t)]x(t) + [D+\Delta D(t)]\omega(t), t\ge 0 \\ x(0) = x_0 \in\mathbb{R}^n \end{cases} \quad (4)$$

Definition 2.5. For given positive numbers c_1, c_2, T_f and a symmetric positive definite matrix *R*, the system (4) is finite-time stable with respect to (c_1, c_2, T_f, R, d) if and only if

$$x_0^TRx_0 \le c_1 \Rightarrow x^T(t)Rx(t) < c_2, \quad t\in[0,T_f],$$

for all disturbances $\omega(t)\in\mathbb{R}^p$ satisfying (2).

Definition 2.6. If there exist a feedback control law $u^*(t) = Kx(t)$ and a positive number J^* such that the closed-loop system

$$\begin{cases} T_t^{\alpha}x(t) = [A+\Delta A(t) + BK + \Delta B(t)K]x(t) \\ \qquad\qquad +[D+\Delta D(t)]\omega(t), t\ge 0 \\ x(0) = x_0\in\mathbb{R}^n \end{cases} \quad (5)$$

is finite-time stable with respect to (c_1, c_2, T_f, R, d) and the cost function (4) satisfies $J(u)\le J^*$ then the value J^* is a guaranteed cost value and the designed control $u^*(t)$ is said to be a guaranteed cost controller.

3. Main Results

The following theorem derives a new sufficient condition for the design of a state feedback controller that makes the closed-loop system (5) is finite-time stable and guarantees an adequate cost level of performance.

Theorem 3.1. *Assume that the conditions (H1) and (H2) are satisfied. For given positive numbers c_1, c_2, T_f and a symmetric positive definite matrix R, if there exist a symmetric positive definite matrix P, a matrix Y with appropriate dimensions and positive scalars $\varepsilon_1, \varepsilon_2$ satisfying the following conditions:*

$$\begin{bmatrix} M_{11} & PH_a^T & Y^T H_b^T & PQ_1 & Y^T Q_2 \\ * & -\varepsilon_1 I & 0 & 0 & 0 \\ * & * & -\varepsilon_2 I & 0 & 0 \\ * & * & * & -Q_1 & 0 \\ * & * & * & * & -Q_2 \end{bmatrix} < 0, \quad (6a)$$

$$\lambda_2 c_1 + \frac{(1+\lambda_{\max}(H_d^T H_d))d}{\alpha} T_f^\alpha < \lambda_1 c_2, \qquad (6b)$$

where

$$M_{11} = AP + PA^T + BY + Y^T B^T + \varepsilon_1 E_a E_a^T + DD^T + E_d E_d^T + \varepsilon_2 E_b E_b^T$$

$$\bar{P} = R^{-\frac{1}{2}} P^{-1} R^{-\frac{1}{2}}, \lambda_1 = \lambda_{\min}(\bar{P}), \lambda_2 = \lambda_{\max}(\bar{P}),$$

then the closed-loop system (5) is finite-time stable with respect to (c_1, c_2, T_f, R, d). *Moreover,* $u(t) = YP^{-1}x(t), \forall t \ge 0,$ *is a guaranteed cost controller for the system (1) and the guaranteed cost value is given by*

$$J^* = \frac{d(1+\lambda_{max}(H_d^T H_d))}{\alpha} T_f^\alpha + \lambda_2 c_1$$

Proof. We consider the following non-negative quadratic function for the closed-loop system (5):

$V(x(t)) = x^T(t)P^{-1}x(t)$. From Lemma 2.4, the conformable fractional derivative of $V(x(t))$ along the solution of the system (5) is defined as

$$\begin{aligned} T^\alpha V(x(t)) &= 2x^T(t)P^{-1}T^\alpha x(t) \\ &= x^T(t)[P^{-1}A + A^T P^{-1} + P^{-1}BK + K^T B^T K^T]x(t) \qquad (7) \\ &+ 2x^T(t)P^{-1}E_a F_a(t) H_a x(t) + 2x^T(t)P^{-1}D\omega(t) \\ &+ 2x^T(t)P^{-1}E_d F_d(t) H_d \omega(t) + 2x^T(t)P^{-1}E_b F_b(t) H_b K x(t) \end{aligned}$$

By using the Cauchy matrix inequality, we have the following estimates

$$\begin{aligned} & 2x^T(t)P^{-1}E_a F_a(t) H_a x(t) \le \\ & \varepsilon_1 x^T(t)P^{-1}E_a F_a(t) P^{-1} H_a x(t) + \varepsilon_1^{-1} x^T(t) H_a^T H_a x(t), \\ & 2x^T(t)P^{-1}D\omega(t) \le x^T(t)P^{-1}DD^T x(t) + \omega^T(t)\omega(t), \quad (8) \\ & 2x^T(t)P^{-1}E_d F_d(t) H_d \omega(t) \\ & \le x^T(t)P^{-1}E_d E_d^T P^{-1} x(t) + \omega^T(t) H_d^T H_d \omega(t) \\ & 2x^T(t)P^{-1}E_b F_b(t) H_b K x(t) \qquad (10) \\ & \le \varepsilon_2 x^T(t)P^{-1}E_b F_b^T(t) P^{-1} x(t) + \varepsilon_2^{-1} x^T(t) K^T H_b^T H_b K x(t). \end{aligned}$$

From (7)-(10), we obtain

$$\begin{aligned} T^\alpha V(x(t)) &= x^T(t)\Omega x(t) + (1+\lambda_{max}(H_d^T H_d)) \|\omega(t)\|^2 \\ &- x^T(t)[Q_1 + K^T Q_2 K]x(t), \qquad (11) \end{aligned}$$

Where

$$\begin{aligned} \Omega &= P^{-1}A + A^T P^{-1} + P^{-1}BK + K^T B^T K^T + \varepsilon_1 P^{-1}E_a E_a^T P^{-1} \\ &+ \varepsilon_1^{-1} H_a^T H_a + P^{-1}DD^T P^{-1} + P^{-1}E_d F_d^T P^{-1} + \varepsilon_2 P^{-1} E_b E_b^T P^{-1} \\ &+ \varepsilon_2^{-1} P^{-1} E_b E_b^T P^{-1} + \varepsilon_2^{-1} K^T H_b^T H_b K + Q_1 + K^T Q_2 K \end{aligned}$$

Now, pre- and post-multiply both sides by P and letting $K = YP^{-1}$, we have

$$\begin{aligned} P\Omega P &= AP + PA^T + BY + B^T Y^T + \varepsilon_1 E_a E_a^T + \varepsilon_1^{-1} PH_a^T H_a P + DD^T \\ &+ E_d F_d^T + \varepsilon_2 E_b E_b^T + \varepsilon_2^{-1} E_b E_b^T + \varepsilon_2^{-1} Y^T H_b^T H_b Y + PQ_1 P + Y^T Q_2 Y \end{aligned}$$

Note that $\Omega < 0$ is equivalent to $P\Omega P < 0$. Using the Schur Complement Lemma, we have that $P\Omega P < 0$ is equivalent to (6a). Therefore, from the conditions (6a), (11) and the fact that $x^T(t)[Q_1 + K^T Q_2 K]x(t)$, $\forall t \ge 0$ we have

$$T^\alpha V(x(t)) \le (1+\lambda_{max}(H_d^T H_d)) \|\omega(t)\|^2 . \qquad (12)$$

Integral with order α both sides of (12) from 0 to $t (0 < t \le T_f)$ and using Lemma 2.3, we obtain

$$\begin{aligned} x^T(t)P^{-1}x(t) &\le x^T(0)P^{-1}x(0) + I^\alpha((1+\lambda_{max}(H_d^T H_d))\omega^T(t)\omega(t) \\ &= x^T(0)P^{-1}x(0) + (1+\lambda_{max}(H_d^T H_d)\int_0^t s^{\alpha-1}\omega^T(s)\omega(s)ds \\ &\le x^T(0)P^{-1}x(0) + d(1+\lambda_{max}(H_d^T H_d)\int_0^t s^{\alpha-1}ds \\ &\le x^T(0)P^{-1}x(0) + \frac{d(1+\lambda_{max}(H_d^T H_d)}{\alpha} T_f^\alpha \qquad (13) \end{aligned}$$

On the other hand, we have

$$\begin{aligned} x^T(t)P^{-1}x(t) &= x^T(t)R^{\frac{1}{2}}\bar{P}R^{\frac{1}{2}}x(t) \\ &\ge \lambda_{\min}(\bar{P})x^T(t)Rx(t) \qquad (14) \\ &= \lambda_1 x^T(t)Rx(t) \end{aligned}$$

and

$$x^T(0)P^{-1}x(0) = x^T(0)R^{\frac{1}{2}}\bar{P}R^{\frac{1}{2}}x(0)$$
$$\le \lambda_{\max}(\bar{P})x^T(0)Rx(0) \qquad (15)$$
$$= \lambda_2 x^T(0)Rx(0) \le \lambda_2 c_1$$

From (13)-(15), we get

$$\lambda_1 x^T(t)Rx(t) \le V(x(t)) = x^T(t)P^{-1}x(t)$$
$$\le \lambda_2 c_1 + \frac{d(1+\lambda_{max}(H_d^T H_d)}{\alpha}T_f^\alpha$$

Condition (6b) implies that $x^T(t)Rx(t) < c_2$. Thus, the system (5) is finite-time stable with respect to (c_1, c_2, T_f, R, d). Next, we will find the guaranteed cost value of the cost function (3). From conditions (6a) and (11), we have the following estimate

$$T^\alpha V(x(t)) = (1+\lambda_{max}(H_d^T H_d))\|\omega(t)\|^2$$
$$- x^T(t)[Q_1 + K^T Q_2 K]x(t), \qquad (16)$$

Integral with order α both sides of (16) from 0 to T_f and using Lemma 2.3, we get

$$V(x(T_f)) - V(x(0)) \le (1+\lambda_{\max}(H_d^T H_d))\int_0^t s^{\alpha-1}\omega^T(s)\omega(s)ds$$
$$- J(u), \qquad (17)$$

Therefore, we have

$$J(u) \le (1+\lambda_{\max}(H_d^T H_d))\int_0^{T_f} s^{\alpha-1}\omega^T(s)\omega(s)ds + V(x(0)),$$
$$\le \lambda_2 c_1 + \frac{d(1+\lambda_{max}(H_d^T H_d)}{\alpha}T_f^\alpha \qquad (18)$$

due to $V(x(T_f)) > 0$, which completes the proof of the theorem.

Remark 2. We have the following procedure which allows us to solve the problem of finite-time guaranteed cost control for uncertain conformable fractional-order system (1) by using Matlab's LMI Control Toolbox:

Step 1. Solve the linear matrix inequality (6a) and obtain a symmetric positive definite matrix *P*, a matrix *Y* and two positive scalars $\varepsilon_1, \varepsilon_2$.

Step 2. Compute the invertible matrix P^{-1}, matrix $\bar{P} = R^{-\frac{1}{2}}P^{-1}R^{-\frac{1}{2}}$ and numbers $\lambda_1 = \lambda_{\min}(\bar{P}),\ \lambda_2 = \lambda_{max}(\bar{P})$.

Step 3. Check condition (6b) in Theorem 3.1. If it holds, enter Step 4; else return to Step 1.

Step 4. The guaranteed cost controller for the system (1) is given by $u(t) = YP^{-1}x(t)$.

Particularly, when $\Delta A(t) \equiv 0, \Delta B(t) \equiv 0, \Delta D(t) \equiv 0$ then system (1) is reduced to the linear conformable fractional order systems

$$\begin{cases} T^\alpha x(t) = Ax(t) + D\omega(t) + Bu(t), t \ge 0 \\ x(0) = x_0 \end{cases} \qquad (19)$$

According to Theorem 3.1, we immediately have the following result.

Corollary 3.1. *For given positive numbers* c_1, c_2, T_f *and a symmetric positive definite matrix R, if there exist a symmetric positive definite matrix P, a matrix Y with appropriate dimensions satisfying the following conditions*

$$\begin{bmatrix} N_{11} & PQ_1 & Y^T Q_2 \\ * & -Q_1 & 0 \\ * & * & -Q_2 \end{bmatrix} < 0, \qquad (20a)$$

$$\lambda_2 c_1 + \frac{d}{\alpha}T_f^\alpha < \lambda_1 c_2, \qquad (20b)$$

where

$$N_{11} = AP + PA^T + BY + Y^T B^T + DD^T$$

$$\bar{P} = R^{-\frac{1}{2}}P^{-1}R^{-\frac{1}{2}}, \lambda_1 = \lambda_{\min}(\bar{P}), \lambda_2 = \lambda_{\max}(\bar{P}),$$

then the closed-loop system is finite-time stable with respect to (c_1, c_2, T_f, R, d).

Moreover, $u(t) = YP^{-1}x(t), \forall t \ge 0$ *is a guaranteed cost controller for the system (20) and the guaranteed cost value is given by*

$$J* = \frac{d}{\alpha}T_f^\alpha + \lambda_2 c_1$$

4. Numerical Example. Consider the system (20), where α=0.96, $x(t) \in \mathbb{R}^2, u(t) \in \mathbb{R}, \omega(t) = 0.2\cos t$, and

$$A=\begin{bmatrix}-0.9 & 2 & 0\\ 0 & 1 & 1.5\\ 0 & 1 & -2\end{bmatrix}, D=\begin{bmatrix}0.1 & 0 & 0\\ 0 & 0.1 & 0\\ 0 & 0 & 0.1\end{bmatrix}, B=\begin{bmatrix}1\\ 2\\ 2\end{bmatrix}$$

We have the disturbance $\omega(t)$ satisfying the condition (2) with d = 0.04. The cost function associated with the considered system is given in (3) with $Q_1=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}, Q_2=[0.1].$

Given $c_1=1,\ c_2=1.7,\ T_f=2, R=I$ we found that the conditions (20a) and (20b) in Corollary 3.1 are satisfied with

$$P=\begin{bmatrix}0.4016 & 0.0249 & -0.0436\\ 0.0249 & 0.3521 & 0.0043\\ -0.0436 & 0.0043 & 0.3575\end{bmatrix},$$

$$Y=\begin{bmatrix}-0.1428 & -0.4762 & 0.0851\end{bmatrix}.$$

By Corollary 3.1, the closed-loop system of the considered system is finite-time stable with respect to (1, 1.7, 2, *I*, 0.04) and the guaranteed cost value is *J** = 0.05141. Moreover

$$u(t)=\begin{bmatrix}-0.2484 & -1.3376 & 0.2238\end{bmatrix}x(t), t\geq 0.$$

The Figure 1, figure 2 show the respone of $x^T(t)Rx(t)$ of the open-loop systems and the closed-loop system. On the figure 3, the response of the control input signal $u(t)Kx(t)$ is shown. We find easily that $x_1(0)=1, x_2(0)=-0.2, x_3(0)=1.$ With this result, it is clear from the Figure 2 that the closed-loop system is finite-time stable with respect to (1, 1.7, 2, *I*, 0.01).

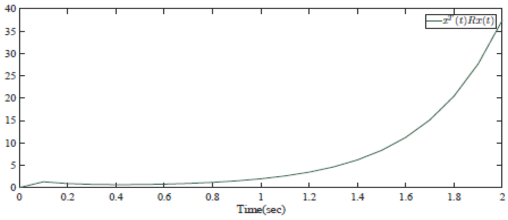

Figure 1. *The response of* $x^T(t)Rx(t)$ *of the open-loop system*

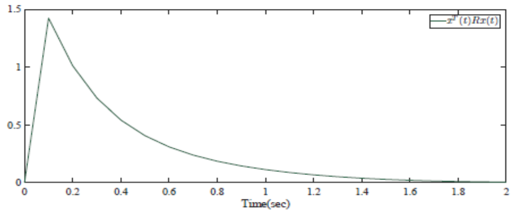

Figure 2. *The response of* $x^T(t)Rx(t)$ *of the closed-loop system*

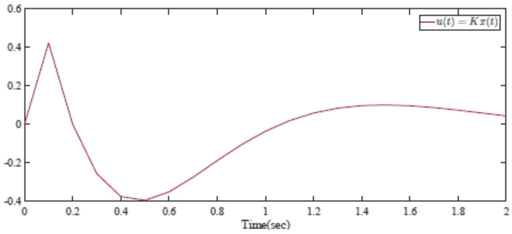

Figure 3. *The response of the control input signal* $u(t)Kx(t)$

5. Conclusion

In this paper, the problem of robust finite-time guaranteed cost control for linear uncertain conformable fractional-order system has been investigated. Based on some well-behaved properties of the conformable fractional calculus and finite-time stability theory, new sufficient conditions for the design of a state feedback controller which makes the closed-loop systems finite-time stable and guarantees an adequate cost level of performance have been derived in term of LMIs. A numerical example has been given to demonstrate the simplicity of our design method.

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