ON THE EFFICIENCY OF PIEZOELECTRIC ENERGY HARVESTER WITH EXPONENTIALLY TAPERED CANTILEVER BEAM

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Abstract: *This paper theoretically examines the efficiency of exponentially tapered cantilever piezoelectric energy harvester which is subjected to base excitation. In anaysis models, the lumped parameters of composite beam such as natural frequency, equivalent stiffness and mass are defined from the continuous model firstly, then they are used in discrete model which is gorvened by electromechanically coupled governing equations. Based on the discrete model, a recently estimation of efficiency is applied to evaluate the transferred energy rate of exponentially tapered cantilever piezoelectric energy harvester with various shape.*

Keywords: exponentially tapered cantilever beam, piezoelectric energy harvester, efficiency.

1. INTRODUCTION

Among kinetic energy harvesting applications for low-power autonomous systems, piezoelectric energy harvester (PEH) is one of the most common generators used to convert mechanical vibrations into electrical energy due to its high energy density (Roundy, et al 2003). An excellent review of piezoelectric energy harvesting techniques developed in the last decade was summerized in (Inman, et al 2018). The main focuses on PEH now are improving the performance via high-performance piezoelectric materials, structure and manufacturing process innovation, optimization of dynamic characteristics (Yang, et al 2017).

In energy flow analysis for piezoelectric energy harvesting systems, efficiency is considered as an important criterion to evaluate the transferred energy rates. There are three major phases/steps in PEH: *mechanical-mechanical energy transfer*, *mechanical-electrical energy conversion*, *electrical-electrical energy transfer* (Uchino, et al 2010; Yang, et al 2017). Throughout PEH, the relative energy loss are mechanical, mechanical-electrical transduction and electrical ones. In (Yang, et al 2017) several studies on efficiency was reviewed and a new estimation of the efficiency was developed. As shown in the analytical expression that efficiency is greatly affected by geometry of PEH, electromechanical coupling effect, damping effect, excitation frequency and electrical load.

In order to improve the performance of PEH, many recent studies have proposed various geometries of PEH mechanical structure to attain higher stress, strain and consequently higher voltage and power from the same piezoelectric material. It has been proven in the literature that some type of tapered cantilever beam can obtain higher energy than the rectangular one from higher excitation frequency (Inman, et al 2018; Hosseini, et al 2015; Udhayakumar, et al 2018). Nevertheless these papers only focused on optimizing the geometrical factors of PEH structrure at resonant state to maximize the input mechanical energy, but without considering the efficiency.

In this paper, we examine the efficiency of PEH with exponentially tapered cantilever beam with the estimation proposed in (Yang, et al 2017). The lumped parameters using in single-degree-of-freedom (SDOF) model is derived from Euler Bernoulli beam model whereas the natural frequency is defined by using Rayleigh–Ritz method.

2. LUMPED PARAMETERS OF EXPONENTIALLY TAPERED CANTILEVER PEH

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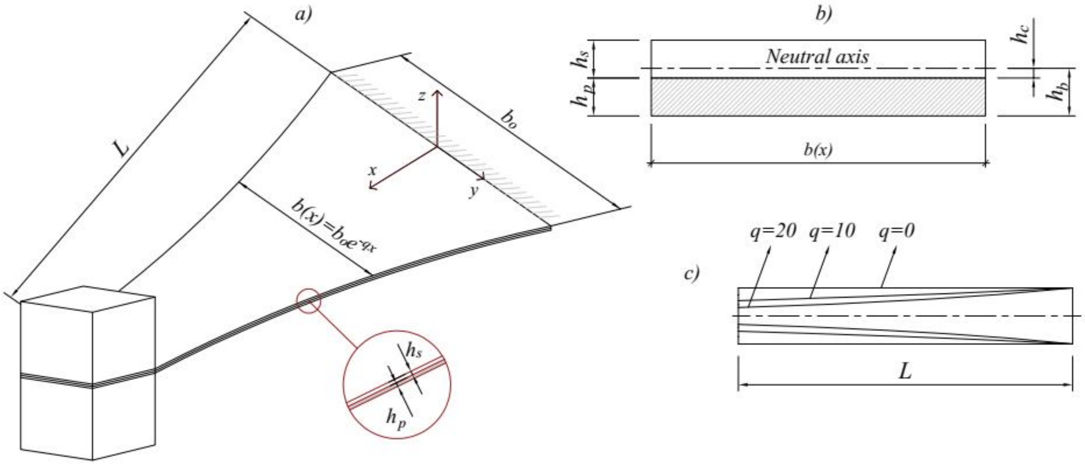

Figure 1. A unimorph exponentially tapered cantilever PEH

Consider a tapered unimorph piezoelectric beam in which beam's width is varying exponentially through the length L with tip mass m_t (Figure 1a) whereas the thickness of piezo layer and substructure are h_p, h_s (Figure 1b) respectively (Kordkheili, et al 2015). The width $b(x)$ and flexural stiffness $E(x)I(x)$ per unit length distributions can be expressed by

$$b(x) = b_0 \exp(-qx); h = h_s + h_p$$
$$m_b = \int_0^L \rho h b(x) dx = (\rho_s h_s + \rho_p h_p) \int_0^L b_0 \exp(-qx) dx; \rho = \frac{\rho_s h_s + \rho_p h_p}{h_s + h_p}$$
$$I(x) = \frac{b(x) h_s^3}{12} + \frac{b(x) h_p (h_p^2 + 3h_p h_s + 3h_s^2)}{12} = I_s(x) + I_p(x) \tag{1}$$
$$E(x) = \frac{E_s I_s(x) + E_p I_p(x)}{I(x)} = \frac{\frac{b(x) h_s^3}{12} E_s + \frac{b(x) h_p (h_p^2 + 3h_p h_s + 3h_s^2)}{12} E_p}{I(x)}$$

where b_0 is width of the composite beam at $x = 0$, m_b is mass, ρ_s, ρ_p, E_s, E_p are the densities and modulus of elasticity of beam structure and piezoelectric materials, respectively. Using Euler-Bernoulli beam theory, the calculated deflection distribution along the beam length subjected to a concentrated load P at the tip is defined from equation $z''(x) = M(x)/E(x)I(x) = P(L-x)/E(x)I(x)$. The result is

$$z(x) = P \frac{e^{2qx}(2Lq + 1 - qx) - xq(2Lq + 1) - (Lq + 1)}{4E(0)I(0)q^3} \tag{2}$$

In order to find the lumped parameters of the PEH beam, the equivalent stiffness k of the beam with and without tip mass are assumed to be the same, and the relation between the force P and the deflection at $x = L$ is

$$P = kz(L) \tag{3}$$

Using (2) and (3), the equivalent stiffness k is calculated as

$$k = \frac{4E(0)I(0)q^3}{e^{2qL} - 2q^2L^2 - 2qL - 1} \tag{4}$$

The deflection function of (2) can be used as the mode shape. Hence the natural frequency ω_0 of the composite beam can be defined by using Rayleigh–Ritz method, $T_{\max} = V_{\max}$, in which the maximum kinetic and potential energies of the system are, respectively

$$T_{\max} = \int_0^L \frac{\omega_0{}^2}{2} \rho h b(x) z(x)^2 \, dx = \omega_0{}^2 P^2 \rho h b_0 e^{-Lq} \left(864 E(0)^2 I(0)^2 q^7\right)^{-1} \times$$
$$\times\Big(17e^{4Lq} + e^{2Lq}\left(54 + 108Lq - 216L^2q^2\right)$$
$$+e^{Lq}\left(64 + 192Lq + 288L^2q^2\right) - 540Lq - 756L^2q^2 - 432L^3q^3 - 108L^4q^4 - 135\Big) \quad (5)$$

$$V_{\max} = \int_0^L \frac{1}{2} E(x) I(x) \left(\frac{\partial^2 z}{\partial x^2}\right)^2 dx = \frac{P^2 4q\left(e^{2Lq} - 2L^2q^2 - 2Lq - 1\right)}{32E(0)I(0)q^4} \quad (6)$$

From (5), (6) one has

$$\omega_0 = 6q^2 \left(\frac{3E(0)I(0)e^{Lq}\left(e^{2Lq} - 2L^2q^2 - 2Lq - 1\right)}{\rho h b_0}\right)^{1/2}$$
$$\times\Big(17e^{4Lq} + e^{2Lq}\left(54 + 108Lq - 216L^2q^2\right) + e^{Lq}\left(64 + 192Lq + 288L^2q^2\right) -$$
$$-540Lq - 756L^2q^2 - 432L^3q^3 - 108L^4q^4 - 135\Big)^{-1/2} \quad (7)$$

Assuming that the natural frequencies defined from continuous and lumped mass models without tip mass are equal and using the equivalent stiffness k defined in (4), one has

$$\omega_0 = \sqrt{\frac{k}{m_{eq}}} \quad (8)$$

where m_{eq} is the equivalent mass. Consequently, $m_{eq} = k / \omega_0{}^2$. Including the tip mass m_t, the lumped mass is

$$m = m_{eq} + m_t = \frac{k}{\omega_0{}^2} + m_t \quad (9)$$

Therefore the equivalent natural frequency is

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{\omega_0^2 k}{k + \omega_0^2 m_t}}$$
$$= 6q^2\left[3E(0)I(0)e^{Lq}\left(e^{2Lq} - 2L^2q^2 - 2Lq - 1\right)\right]^{1/2}\left(\rho h b_0\right)^{-1/2} \times$$
$$\times\Big[17e^{4Lq} + e^{2Lq}\left(54 + 108Lq - 216L^2q^2\right) + e^{Lq}\left(64 + 192Lq + 288L^2q^2\right) -$$
$$-540Lq - 756L^2q^2 - 432L^3q^3 - 108L^4q^4 - 135 +$$
$$+27qm_t e^{Lq}\left(e^{2Lq} - 2L^2q^2 - 2Lq - 1\right)^2\Big]^{-1/2} \quad (10)$$

It is worth to note that when $q \to 0$ then $b(L) \to b_0$, the exponentially tapered cantilever beam becomes to the rectangular one, and

$$\lim_{q\to 0} k = \frac{3E(0)I(0)}{L^3}; \lim_{q\to 0} \omega_0 = 1.875^2 \sqrt{\frac{E(0)I(0)}{\rho b_0 h L^4}}; \lim_{q\to 0} m = \frac{33}{140}\rho b_0 h L + m_t \quad (11)$$

which is coincided the proven results in the literature (Erturk, et al 2011; Udhayakumar, et al 2018).

3. SDOF MODEL OF UNIMORPH CANTILEVER PEH

The SDOF dynamic system of a unimorph cantilever PEH with lumped mass subjected to base excitation can be expressed by (Figure)

$$m\ddot{X} + c\dot{X} + kX + \theta V = -m\ddot{Z} \qquad (12)$$

$$C_p\dot{V} + \frac{1}{R}V = \theta\dot{X} \qquad (13)$$

where m, c, k, C_p, R, θ, V, X respectively are the equivalent mass, damping, stiffness, piezoelectric internal capacitance, external resistance, effective electromechanical coupling coefficient, output voltage over piezo element, relative displacement, and the base excitation Z is gorvened by

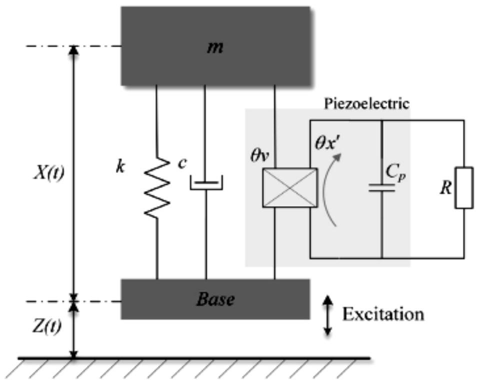

Figure 2. Schematic of a piezoelectric energy harvesting system (Yang, et al 2017)

$$\ddot{Z} = \omega_n^2 A\cos(\omega t) \qquad (14)$$

Let us denote

$$\tilde{\omega} = \frac{\omega}{\omega_n} = \omega\sqrt{m/k} \qquad (15)$$

$$\xi = \frac{c}{2m\omega_n}; \kappa^2 = \frac{\theta^2}{kC_p}; \alpha = \frac{1}{RC_p\omega_n}; \tau = \omega_n t; r = \frac{1}{RC_p\omega} = \frac{\alpha}{\tilde{\omega}} \qquad (16)$$

Physically ω is excitation frequency; $\tilde{\omega}$ is frequency ratio; ξ is the damping ratio, κ^2 is the electromechanical coupling coefficient, r is resistance ratio. In (Yang, et al 2017) efficiency is defined by the ratio of the net output electrical energy E_{out} and the net input mechanical energy W in which the formulation is

$$\eta = \frac{E_{out}}{W} = \frac{\kappa^2}{2\xi\tilde{\omega}\left(\frac{1}{r} + r\right) + \kappa^2} = \frac{\kappa^2}{2\frac{\xi}{\alpha}\tilde{\omega}^2 + 2\xi\alpha + \kappa^2} \qquad (17)$$

Obviously $\eta < 1$ due to energy loss. To get higher output electrical energy, a fixed value load resistance $R = 1/C_p\omega_n$ is selected (Yang, et al 2017). It leads to:

$$\alpha = 1 \qquad (18)$$

$$\eta = \frac{\kappa^2}{2\xi\left(\tilde{\omega}^2 + 1\right) + \kappa^2} \qquad (19)$$

Eq.(19) shows that the efficiency is not largest at $\tilde{\omega} = 1$ while the input mechanical energy get highest values at resonant state. It is emphasised in examination of a certain rectangular cantilever PEH (Yang, et al 2017) that conditions to attain the maximum power transfer (around resonant point, $\tilde{\omega} = 1$) do not coincide with conditions to achieve the highest energy conversion efficiency.

4. EXAMINING EFFICIENCY OF AN EXPONENTIALLY TAPERED CANTILEVER PEH

Consider an exponentially tapered cantilever PEH with geometric and material properties as shown in Table 1. The piezoelectric material is PZT-5A, the substructure material is aluminium. To calculate the efficiency in Eq.(19), the lumped parameters as equivalent stiffness k, the equivalent mass m, and the natural frequency ω_n is calculated from is defined by Eq.(4), (7)-(10), respectively. In (16) the equivalent piezoelectric internal capacitance C_p and effective electromechanical coupling coefficient θ are defined by

$$C_p = \frac{1}{L}\int_0^L b(x)dx\frac{L}{h_p}\varepsilon_{33}^S = \frac{b_0\left(1-e^{-qL}\right)}{qh_p}\varepsilon_{33}^S$$
$$\theta = \frac{1}{L}\int_0^L b(x)dx\frac{(h_b^2-h_c^2)}{2h_pL}e_{31} = \frac{(h_b^2-h_c^2)b_0\left(1-e^{-qL}\right)}{2h_pL^2q}e_{31} \quad (20)$$

where h_b and h_c denote the positions of the bottom and the top of the piezoelectric layer from the neutral axis, respectively (Figure 1 b) (Erturk, et al 2011). In (20), the integrations are mean values of $b(x)$.

Table 1. Geometric and material properties of exponentially tapered cantilever PEH

Name	*Value*	*Name*	*Value*
Length (L)	73 mm	Piezo layer thickness (h_p)	0.5 mm
Width at fixed end (b_0)	21 mm	Piezo density (ρ_p)	7800 kg/m^3
Substructure thickness (h_s)	0.5 mm	Piezo Young's modulus (E_p)	66 GPa
Substructure Young's modulus (E_s)	65 GPa	Stress constant (e_{31})	-11.5
Substructure density (ρ_s)	2730 kg/m^3	Vacuum permittivity (ε_0)	8.854 x 10^{-12}
Tipp mass (m_t)	15.6 g	Absolute permittivity $\left(\varepsilon_{33}^S\right)$	1500ε_0

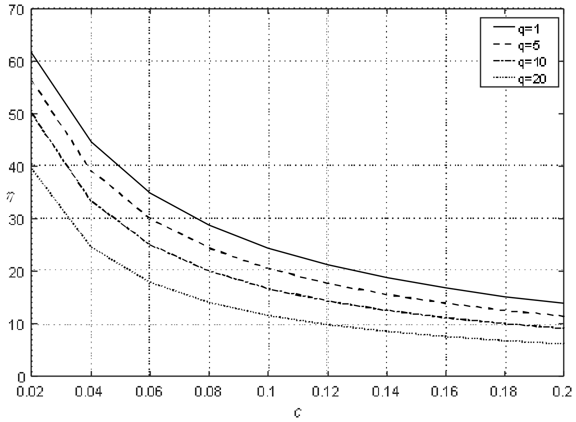

Figure 3. Effect of the damping on efficiency (α=1, ϖ=1)

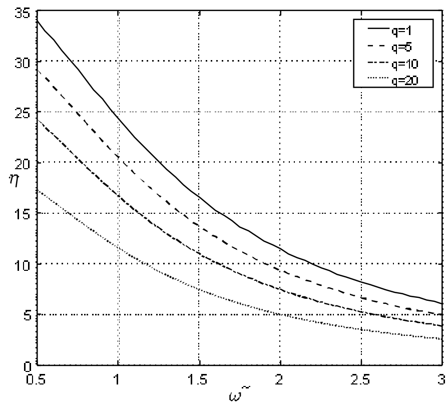

Figure 4. Effect of the frequency ratio on efficiency (α=1, c=0.1)

Figure 3 shows the small damping effect of the structural on efficiency in resonant state. For various shape of exponentially tapered cantilever, the efficiency decreases small as damping increases, that means the efficiency value tends to keep constant, when damping turns strong. It is shown that the lower value of q is, the higher efficiency is obtained. When c is very small, the efficiency in case of *q=1* reaches to 60% in comparing with 40% in case of *q=20*. Figure shows the effect of the frequency ratio on efficiency. Similar as the effect of damping, the lower value of q provides higher efficiency. The efficiency value drops very fast when the excitation frequency rises, for example just about 5% as $\tilde{\omega} > 1.5$. We can found this important characteristic in (Yang, et al 2017). Therefore the preferred consideration is to design PEH working in the resonant region $\tilde{\omega} = [0.9,1]$.

5. CONCLUSIONS

In this paper, we theoretically studied efficiency of an exponentially tapered cantilever PEH subjected to base excitation. Firstly, analytical expression of lumped parameters of the composite beam such as equivalent stiffness, is defined based on Euler-Bernoulli beam model, the natural frequency is defined by using Rayleigh–Ritz method, the equivalent mass is calculated from equivalent stiffness and natural frequency. The lumped parameters then are used for analysing the efficiency of the PEH following Erturk'estimation. The application for a certain exponentially tapered cantilever PEH shows that the main characteristics agree with available result extracted from study of rectangular cantilever PEH. Using the approach in this paper, efficiency of other PEH structures, such as inverted taper in width configuration, may be analysed to archieved optimum design.

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Tóm tắt:

HIỆU SUẤT CỦA THIẾT BỊ KHAI THÁC NĂNG LƯỢNG ÁP ĐIỆN KIỂU DẦM CÔNG XÔN THON CÓ BIÊN DẠNG LÀ HÀM MŨ

Bài báo nghiên cứu hiệu suất của thiết bị khai thác năng lượng kiểu áp điện (PEH) dạng dầm công xôn thon có biên dạng là hàm mũ chịu kích động nền. Hiệu suất của thiết bị được phân tích bằng mô hình một bậc tự do với liên kết cơ-điện, trong đó các thông số động lực học như tần số tự nhiên, độ cứng và khối lượng tương đương được xác định từ mô hình dầm liên tục. Biểu thức hiệu suất của Yang và cộng sự được sử dụng để đánh giá hiệu suất của PEH với các biên dạng khác nhau.

Từ khóa: dầm công xôn thon, thiết bị khai thác năng lượng kiểu áp điện, hiệu suất.

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